CPSC 314
Midterm 1

October 11, 2019

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: ____________________________________________________________

Student Number: ________________________________________________

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1. Coordinate Frames

(a) (3 points) Express point \( P \) in each of the three coordinate frames.

\[
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}_W = \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}_A
\]

(b) (3 points) Express vector \( V \) in each of the three coordinate frames.

\[
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}_W = \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}_A
\]

(c) (2 points) Find the \( 3 \times 3 \) affine transformation matrix which takes a point from \( F_A \) and expresses it in terms of \( F_W \). I.e., determine \( M \), where \( P_W = MP_A \). Write your answer in the space to the right of the diagram above.

(d) (2 points) Find the \( 3 \times 3 \) affine transformation matrix which takes a point from \( F_A \) and expresses it in terms of \( F_B \). I.e., determine \( M \), where \( P_B = MP_A \). Write your answer in the space to the right of the diagram above.

(e) (2 points) Develop a sequence of rotations, translates, and scales to construct the same \( 3 \times 3 \) affine transformation as in part (c), i.e., \( P_W = MP_A \). Express your solution as an algebraic composition of transformation matrices, in whatever order your prefer, e.g., \( P_W = \text{Trans}(-1, 2, 0)\text{Rot}(z, 45^\circ)\text{Scale}(2, 3, 1)P_A \).
2. Composition of Transformations
Consider the house which is shown below in its untransformed state, e.g., $F_h = F_W$, and the following three.js code:

```javascript
house.matrix.identity();
house.matrix.multiply(new THREE.Matrix4().makeRotationZ(-Math.PI/2)); // step A
house.matrix.multiply(new THREE.Matrix4().makeTranslation(-2,4,0)); // step B
house.matrix.multiply(new THREE.Matrix4().makeScale(2,1,1)); // step C
```

(a) (3 points) Sketch all the intermediate and final transformations of the house for the transformations applied to `house.matrix`. Label them A,B,C. Draw the final coordinate frame, and label it with $F_C$.

(b) (1 point) Give an algebraic expression for `house.matrix` after the steps above, i.e., a transformation that would take a point from $F_C$ to $F_W$. Use `Rot(z,\theta)`, `Trans(x,y,z)`, `Scale(sx,sy,sz)` to represent the matrices in your expression.

(c) (1 point) Similarly, give an algebraic expression for the inverse of `house.matrix`, i.e., a transformation that would take a point from $F_W$ to $F_C$. 

![Diagram showing transformations](image)
(d) (3 points) Sketch the coordinate frame and house that would result from running the following lines of code *after* the code given in part (a).

\[
M2 = \text{new THREE.Matrix4().set(}
1, 0, 0, 2,
1, 1, 0, 3,
0, 0, 1, 0,
0, 0, 0, 1);
\]

\[
\text{house.matrix.multiply(M2); // step D}
\]

(e) (3 points) Give \(4 \times 4\) transformation matrices that perform the following:

\[
\text{Trans}(-2, 2, 1) \quad \text{Scale}(2, 1, 2) \quad \text{Rot}(z, -90)
\]

3. (8 points) True/False and short answer

T/F (true or false):

(a) ______ With a pinhole camera model, objects are always in focus.

(b) ______ In a right-handed coordinate system, \(k \times i = j\).

(c) ______ An object is located at a distance \(d\) from the eye, and rendered using perspective projection. If the distance becomes \(3 \times d\), the rendered object will be \(3\times\) larger in the image.

(d) ______ For a perspective view volume, if the value of near is tripled, and top, bot, left, right, far all remain unchanged, then the rendered object will become \(3\times\) larger.

(e) The point \(P_{VCS}(4, 4, -6)\) projects to a point \(P'\) on an image plane placed at \(z_{VCS} = -2\). Give the \(x, y, z\) coordinates of \(P'\). ________

(f) Express the point \((x, y, z, h) = (-8, 2, 3, 3)\) in Cartesian coordinates. ________

(g) Given modeling, viewing, and projection matrices, \(M_{model}, M_{view}, M_{proj}\), and a point \(P_{obj}\), give algebraic expressions that compute:

(i) \(P_{WCS}\); (ii) \(P_{VCS}\); and (iii) \(P_{CCS}\).
4. Scene Graphs

(a) (2 points) In the space above, sketch a scene graph for the simple character model. Place the world frame, $F_W$, at the root of your scene graph. Label the transformations involved, using $M_i$ to designate the matrix, and use arrows to indicate the direction of the change-of-basis transformation, i.e., an arrow from $F_A$ to $F_B$ indicates that the given matrix transforms a point expressed in coordinate frame $F_A$ to coordinate frame $F_B$.

(b) (1 point) Give an algebraic expression for the compound transformation that transforms a point from frame $F_4$ to world coordinates, $F_W$.

(c) (1 point) Give an algebraic expression for the compound transformation that transforms a point from frame $F_2$ to forearm coordinates, $F_6$.

(d) (2 points) Assume that the body has a width of 3 units and a height 5 units. Give an approximate sequence of translate and rotate commands that would correspond to $M_3$, i.e., locating $F_3$ with respect to $F_1$, for the position and orientation shown in the above diagram.
5. Projection Transformations

(a) (2 points) Sketch a one-point perspective projection of a cube.

(b) (3 points) Sketch a side view, i.e., $yz$-plane, of a perspective view volume defined by near=2, far=10, top=1, bot=-1, left=-1, right=1. Shade the region where objects will be visible.

(c) (2 points) The point $P_{VCS}(1,0,-3)$, given in viewing coordinates, is transformed using the projection matrix given below, and onto a 1000 $\times$ 1000 display window. What is the final display location (DCS) of this point? Show your work. Sketch the resulting location in DCS as well as giving a numerical answer.

$$
M_{proj} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -5/3 & -8/3 \\
0 & 0 & -1 & 0
\end{bmatrix},
$$
(d) (2 points) Give another VCS point that would project to the same display location.

(e) (3 points) A simple way to draw the shadow of a polygon is to use a transformation matrix $M$ that computes the projected shadow location, $P'(x', y', z')$, for each polygon vertex $P(x, y, z)$, according to $P' = MP$. A dark-colored “shadow polygon” can then be drawn using the projected vertices. This is illustrated below, for a given lighting direction, $L$, and a ground plane, $y = 0$.

First, use similar triangles to develop an equation for $x'$ as a function of $P(x, y, z)$ and $L(L_x, L_y, L_z)$. Then develop a similar expression for $z'$. Last, develop the matrix $M$ that implements the desired parallel projection.