CPSC 320: Intermediate Algorithm Design	2019/20 Term 2

Tutorial 10

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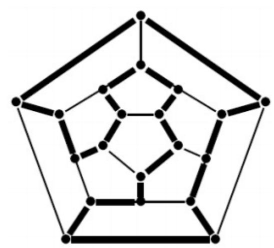
1 Hamiltonian Paths and Bounded-Degree Spanning Trees

The Hamiltonian Path Problem is defined as follows. In a graph $G = (V, E)$, a path that contains *every vertex exactly once* is called a Hamiltonian Path. The computational problem is "Given a graph G, decide if G contain a Hamiltonian Path"

The Hamiltonian Path Problem originates with a puzzle called the icosian game created by Sir William Rowan Hamilton.



A generalization called the Traveling Salesman Problem has many interesting applications, which you might like to read about on this webpage.

The **Bounded-Degree Spanning Tree Problem** is defined as follows. In a graph $G = (V, E)$, a spanning tree T has maximum degree at most d if every vertex in V has at most d incident edges that belong to T. The computational problem is "Given a graph G and an integer d, decide if G contains a spanning tree of maximum degree d."

The motivation for this problem is quite natural. If we wanted to form connections between many networked computers, a natural scheme is to form a spanning tree, because this uses the minimum number of connections to ensure that any computer can reach any other using a path of connections. However, if some computer had too many direct connections, it might get overloaded. So we may be interested to find a spanning tree where each computer has at most d connections.

1. Give a reduction from the Hamiltonian Path problem to the Bounded-Degree Spanning Tree Problem.

2. Assume that there is *no polynomial-time algorithm* for the Hamiltonian Path problem. (This is true, assuming $P \neq NP$. See Kleinberg-Tardos Section 8.5 or Erickson 12.11, where it is proven that the Hamiltonian *Cycle* problem is NP-hard.)

   Prove that there is *no polynomial-time algorithm* for the Bounded Degree Spanning Tree problem.

2 Independent Set and Clique

The **independent set problem** is defined as follows. (See Kleinberg-Tardos Section 8.1 or Erickson Section 12.7). In a graph $G = (V, E)$, a set of vertices S is independent if no two vertices in S are joined by an edge. The problem is "Given a graph G and a number k, does G contain an independent set of size at least k?"

The **clique problem** is as follows. In a graph $G = (V, E)$, a set of vertices S is called a clique if every pair of vertices in S are joined by an edge. The problem is "Given a graph G and a number k, does G contain a clique of size at least k?"

1. Prove IndependentSet $\leq_P$ Clique.

2. Assume that there is *no polynomial-time algorithm* for the Independent Set problem. (This is true, assuming $P \neq NP$. See Kleinberg-Tardos Section 8.2 or Erickson 12.7.)

   Prove that there is *no polynomial-time algorithm* for the Clique problem.