

Tutorial 2

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1. (Algebraic properties of Big-O)

- **(Pointwise Maximum)** Let $h = \max\{f, g\}$, i.e., $h(x) = \max\{f(x), g(x)\}$ for all x . Prove that $h = \Theta(f + g)$.
- **(Transitivity)** Prove that: if $h = O(g)$ and $g = O(f)$, then $h = O(f)$. (Same for Ω and Θ .)
- **(Sums)** Let k be a fixed constant. Let $f_1, \dots, f_k$ and h be functions such that $f_i = O(h)$ for all i . Prove that $f_1 + \dots + f_k = O(h)$. (Same for Ω and Θ .)

2. (Familiar friends)

- **(Polynomials)** Let $T(n) = a_0 + a_1n + \dots + a_dn^d$, with $a_d > 0$. Then $T(n) = \Theta(n^d)$.
- **(Base of log is irrelevant)** Prove that $\log_a n = \Omega(\log_b n)$ for all constants $a, b > 1$.

3. Rank the following functions by order of growth. That is, order them so that

$$f_1 \in O(f_2), \quad f_2 \in O(f_3), \quad \text{etc.}$$

Make sure to indicate whether or not $f_i \in \Theta(f_{i+1})$. For instance you could use the notation

$$n < n^2 = 2n^2 < n^4 \dots$$

Here are the functions to be ordered:

$n^3 - n$	$n!$	$\log(\log n)$	1.01^n
$\sqrt{n \log n}$	17	$4913n^{289}$	3^{2n-17}
$1.01^{1.01^n}$	$\sqrt{n}$	3^n	n^n
$n^{18/17}$	$n \log^{17} n$	3^{2n}	$n^3 + \log n$

4. Let k be a fixed constant (independent of n). Prove that

$$1^k + 2^k + \dots + (n-1)^k + n^k \in \Theta(n^{k+1})$$

Hint: do not use induction. Instead, do O and Ω separately.

Hint: To handle Ω , you will need to use a small trick.