CPSC 320: Intermediate Algorithm Design 2019/20 Term 2

Tutorial 5

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1. **(Finding Recurrence from Code)** Consider the following recursive procedure.

```
procedure TurtleSort(A, first, last)
    if last − first ≤ 2 then
        if last − first = 1 then
            if A[first] > A[first + 1] then
                swap(A[first], A[first + 1])
            end if
        end if
        if last − first = 2 then
            if A[last − 1] > A[last] then
                swap(A[last − 1], A[last])
            end if
            if A[first] > A[first + 1] then
                swap(A[first], A[first + 1])
            end if
        end if
    else
        q1 ← ⌊(3 · first + last)/4⌋
        q2 ← ⌊(first + last)/2⌋
        q3 ← ⌊(first + 3 · last)/4⌋
        TurtleSort(A, first, q2)
        TurtleSort(A, q1, q3)
        TurtleSort(A, q2, last)
        TurtleSort(A, q1, q3)
        TurtleSort(A, first, q2)
        TurtleSort(A, q1, q3)
    end if
end procedure
```

(a): Give a recurrence relation for the running time of the procedure as a function of $n = last - first + 1$. To simplify your answer, do not use floors/ceilings in your recurrence.

(b): Give a Big-O (or actually Big-Θ) bound on the solution of your recurrence from part (a).

2. **(Exponential-Type Recurrence Relations)** Consider the following recurrence relation.

$$T(n) = \begin{cases} 2T(n-1) + 8T(n-2) + n & \text{if } n \geq 3 \\ 7 & \text{if } n = 2 \\ 2 & \text{if } n = 1 \end{cases}$$

We would like to prove that $T(n) = O(4^n)$ using the guess-and-verify method. Unfortunately we will see that the simplest guess will not work. We will use a trick called "strengthening the hypothesis" to allow the guess-and-verify method to work.

(a): **A simple guess.** Let c be an arbitrary constant. Try to prove by induction that $T(n) \leq c4^n$. You should see that this approach does not work.

(b): **Strengthening The Hypothesis:** Let's try to prove by induction the bound $T(n) \leq c4^n - dn$, where c and d are positive real numbers (to be chosen later). Since this bound is smaller than in part (a), it seems that this proof should be even harder. Counterintuitively, it turns out to be easier.