Essentials of Compilation
An Incremental Approach

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This book is dedicated to the programming language wonks at Indiana University.
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The tradition of compiler writing at Indiana University goes back to research and courses about programming languages by Daniel Friedman in the 1970’s and 1980’s. Dan had conducted research on lazy evaluation [Friedman and Wise, 1976] in the context of Lisp [McCarthy, 1960] and then studied continuations [Felleisen and Friedman, 1986] and macros [Kohlbecker et al., 1986] in the context of the Scheme [Sussman and Jr., 1975], a dialect of Lisp. One of the students of those courses, Kent Dybvig, went on to build Chez Scheme [Dybvig, 2006], a production-quality and efficient compiler for Scheme. After completing his Ph.D. at the University of North Carolina, Kent returned to teach at Indiana University. Throughout the 1990’s and 2000’s, Kent continued development of Chez Scheme and taught the compiler course.

The compiler course evolved to incorporate novel pedagogical ideas while also including elements of effective real-world compilers. One of Dan’s ideas was to split the compiler into many small “passes” so that the code for each pass would be easy to understood in isolation. (In contrast, most compilers of the time were organized into only a few monolithic passes for reasons of compile-time efficiency.) Kent, with later help from his students Dipanwita Sarkar and Andrew Keep, developed infrastructure to support this approach and evolved the course, first to use micro-sized passes and then into even smaller nano passes [Sarkar et al., 2004, Keep, 2012]. Jeremy Siek was a student in this compiler course in the early 2000’s, as part of his Ph.D. studies at Indiana University. Needless to say, Jeremy enjoyed the course immensely!

One of Jeremy’s classmates, Abdulaziz Ghuloum, observed that the front-to-back organization of the course made it difficult for students to understand the rationale for the compiler design. Abdulaziz proposed an incremental approach in which the students build the compiler in stages; they start by implementing a complete compiler for a very small subset of the input language, then in each subsequent stage they add a feature to the
input language and add or modify passes to handle the new feature [Ghuloum, 2006]. In this way, the students see how the language features motivate aspects of the compiler design.

After graduating from Indiana University in 2005, Jeremy went on to teach at the University of Colorado. He adapted the nano pass and incremental approaches to compiling a subset of the Python language [Siek and Chang, 2012]. Python and Scheme are quite different on the surface but there is a large overlap in the compiler techniques required for the two languages. Thus, Jeremy was able to teach much of the same content from the Indiana compiler course. He very much enjoyed teaching the course organized in this way, and even better, many of the students learned a lot and got excited about compilers.

Jeremy returned to teach at Indiana University in 2013. In his absence the compiler course had switched from the front-to-back organization to a back-to-front organization. Seeing how well the incremental approach worked at Colorado, he started porting and adapting the structure of the Colorado course back into the land of Scheme. In the meantime Indiana had moved on from Scheme to Racket, so the course is now about compiling a subset of Racket (and Typed Racket) to the x86 assembly language. The compiler is implemented in Racket 7.1 [Flatt and PLT, 2014].

This is the textbook for the incremental version of the compiler course at Indiana University (Spring 2016 - present) and it is the first open textbook for an Indiana compiler course. With this book we hope to make the Indiana compiler course available to people that have not had the chance to study in Bloomington in person. Many of the compiler design decisions in this book are drawn from the assignment descriptions of Dybvig and Keep [2010]. We have captured what we think are the most important topics from Dybvig and Keep [2010] but we have omitted topics that we think are less interesting conceptually and we have made simplifications to reduce complexity. In this way, this book leans more towards pedagogy than towards the absolute efficiency of the generated code. Also, the book differs in places where we saw the opportunity to make the topics more fun, such as in relating register allocation to Sudoku (Chapter 3).

Prerequisites

The material in this book is challenging but rewarding. It is meant to prepare students for a lifelong career in programming languages. We do not recommend this book for students who want to dabble in programming
languages.

The book uses the Racket language both for the implementation of the compiler and for the language that is compiled, so a student should be proficient with Racket (or Scheme) prior to reading this book. There are many other excellent resources for learning Scheme and Racket [Dybvig, 1987, Abelson and Sussman, 1996, Friedman and Felleisen, 1996, Felleisen et al., 2001, 2013, Flatt et al., 2014]. It is helpful but not necessary for the student to have prior exposure to x86 (or x86-64) assembly language [Intel, 2015], as one might obtain from a computer systems course [Bryant and O’Hallaron, 2005, 2010]. This book introduces the parts of x86-64 assembly language that are needed.

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Preliminaries

In this chapter, we review the basic tools that are needed for implementing a compiler. We use abstract syntax trees (ASTs), which refer to data structures in the compilers memory, rather than programs as they are stored on disk, in concrete syntax. ASTs can be represented in many different ways, depending on the programming language used to write the compiler. Because this book uses Racket (http://racket-lang.org), a descendant of Lisp, we use S-expressions to represent programs (Section 1.1). We use grammars to defined programming languages (Section 1.2) and pattern matching to inspect individual nodes in an AST (Section 1.3). We use recursion to construct and deconstruct entire ASTs (Section 1.4). This chapter provides an brief introduction to these ideas.

1.1 Abstract Syntax Trees and S-expressions

The primary data structure that is commonly used for representing programs is the abstract syntax tree (AST). When considering some part of a program, a compiler needs to ask what kind of part it is and what sub-parts it has. For example, the program on the left, represented by an S-expression, corresponds to the AST on the right.
We shall use the standard terminology for trees: each circle above is called a *node*. The arrows connect a node to its *children* (which are also nodes). The top-most node is the *root*. Every node except for the root has a *parent* (the node it is the child of). If a node has no children, it is a *leaf* node. Otherwise it is an *internal* node.

Recall that an *symbolic expression* (S-expression) is either

1. an atom, or
2. a pair of two S-expressions, written \((e_1, e_2)\), where \(e_1\) and \(e_2\) are each an S-expression.

An *atom* can be a symbol, such as *hello*, a number, the null value '\()', etc. We can create an S-expression in Racket simply by writing a backquote (called a quasi-quote in Racket), followed by the textual representation of the S-expression. It is quite common to use S-expressions to represent a list, such as \(a, b, c\) in the following way:

\('(a . (b . (c . ())))'\)

Each element of the list is in the first slot of a pair, and the second slot is either the rest of the list or the null value, to mark the end of the list. Such lists are so common that Racket provides special notation for them that removes the need for the periods and so many parenthesis:

\('(a b c)'\)

For another example, an S-expression to represent the AST \((1.1)\) is created by the following Racket expression:

\('(+ (read) (- 8))'\)

When using S-expressions to represent ASTs, the convention is to represent each AST node as a list and to put the operation symbol at the front of the list. The rest of the list contains the children. So in the above case, the root
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An AST node has operation ‘+’ and its two children are ‘(read)’ and ‘(- 8)’, just as in the diagram (1.1).

To build larger S-expressions one often needs to splice together several smaller S-expressions. Racket provides the comma operator to splice an S-expression into a larger one. For example, instead of creating the S-expression for AST (1.1) all at once, we could have first created an S-expression for AST (1.5) and then spliced that into the addition S-expression.

```
(define ast1.4 ‘(- 8))
(define ast1.1 ‘(+ (read) ,ast1.4))
```

In general, the Racket expression that follows the comma (splice) can be any expression that computes an S-expression.

When deciding how to compile program (1.1), we need to know that the operation associated with the root node is addition and that it has two children: read and a negation. The AST data structure directly supports these queries, as we shall see in Section 1.3 and hence is a good choice for use in compilers. In this book, we will often write down the S-expression representation of a program even when we really have in mind the AST because the S-expression is more concise. We recommend that, in your mind, you always think of programs as abstract syntax trees.

1.2 Grammars

A programming language can be thought of as a set of programs. The set is typically infinite (one can always create larger and larger programs), so one cannot simply describe a language by listing all of the programs in the language. Instead we write down a set of rules, a grammar, for building programs. We shall write our rules in a variant of Backus-Naur Form (BNF) [Backus et al., 1960; Knuth, 1964]. As an example, we describe a small language, named \( R_0 \), of integers and arithmetic operations. The first rule says that any integer is an expression, \( exp \), in the language:

\[
exp ::= int
\]  

Each rule has a left-hand-side and a right-hand-side. The way to read a rule is that if you have all the program parts on the right-hand-side, then you can create an AST node and categorize it according to the left-hand-side. A name such as \( exp \) that is defined by the grammar rules is a non-terminal. The name \( int \) is also a non-terminal, however, we do not define \( int \) because the reader already knows what an integer is. Further, we make the
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simplifying design decision that all of the languages in this book only handle machine-representable integers. On most modern machines this corresponds to integers represented with 64-bits, i.e., the in range \(-2^{63}\) to \(2^{63} - 1\). However, we restrict this range further to match the Racket fixnum datatype, which allows 63-bit integers on a 64-bit machine.

The second grammar rule is the read operation that receives an input integer from the user of the program.

\[
exp ::= (\text{read})
\]

(1.3)

The third rule says that, given an exp node, you can build another exp node by negating it.

\[
exp ::= (\neg exp)
\]

(1.4)

Symbols such as \(-\) in typewriter font are terminal symbols and must literally appear in the program for the rule to be applicable.

We can apply the rules to build ASTs in the \(R_0\) language. For example, by rule (1.2), \(8\) is an exp, then by rule (1.4), the following AST is an exp.

\[
(- 8)
\]

(1.5)

The following grammar rule defines addition expressions:

\[
exp ::= (+ exp exp)
\]

(1.6)

Now we can see that the AST [1.1] is an exp in \(R_0\). We know that (read) is an exp by rule (1.3) and we have shown that \((-8)\) is an exp, so we can apply rule (1.6) to show that \((+ (\text{read}) (- 8))\) is an exp in the \(R_0\) language.

If you have an AST for which the above rules do not apply, then the AST is not in \(R_0\). For example, the AST \((- (\text{read}) (+ 8))\) is not in \(R_0\) because there are no rules for + with only one argument, nor for - with two arguments. Whenever we define a language with a grammar, we implicitly mean for the language to be the smallest set of programs that are justified by the rules. That is, the language only includes those programs that the rules allow.

The last grammar rule for \(R_0\) states that there is a program node to mark the top of the whole program:

\[
R_0 ::= (\text{program} \ exp)
\]
1.3. PATTERN MATCHING

\[
\begin{aligned}
    \text{exp} & ::= \text{int} \mid \text{(read)} \mid (- \text{exp}) \mid (+ \text{exp} \text{exp}) \\
    R_0 & ::= (\text{program exp})
\end{aligned}
\]

Figure 1.1: The syntax of $R_0$, a language of integer arithmetic.

The \texttt{read-program} function provided in \texttt{utilities.rkt} reads programs in from a file (the sequence of characters in the concrete syntax of Racket) and parses them into the abstract syntax tree. The concrete syntax does not include a \texttt{program} form; that is added by the \texttt{read-program} function as it creates the AST. See the description of \texttt{read-program} in Appendix 12.2 for more details.

It is common to have many rules with the same left-hand side, such as $\text{exp}$ in the grammar for $R_0$, so there is a vertical bar notation for gathering several rules, as shown in Figure 1.1. Each clause between a vertical bar is called an alternative.

1.3 Pattern Matching

As mentioned above, one of the operations that a compiler needs to perform on an AST is to access the children of a node. Racket provides the \texttt{match} form to access the parts of an S-expression. Consider the following example and the output on the right.

\[
\text{(match ast1.1}
\]
\[
\quad ['(,op,child1,child2)
\]
\[
\quad (\text{print op})\text{ (newline)}
\]
\[
\quad (\text{print child1})\text{ (newline)}
\]
\[
\quad (\text{print child2})]}
\]
\[
\quad '+
\]
\[
\quad '\text{(read)}
\]
\[
\quad '(- 8)
\]
\[
\text{)}
\]

The \texttt{match} form takes AST (1.1) and binds its parts to the three variables \texttt{op}, \texttt{child1}, and \texttt{child2}. In general, a match clause consists of a \texttt{pattern} and a \texttt{body}. The pattern is a quoted S-expression that may contain pattern-variables (each one preceded by a comma). The pattern is not the same thing as a quasiquote expression used to \texttt{construct} ASTs, however, the similarity is intentional: constructing and deconstructing ASTs uses similar syntax. While the pattern uses a restricted syntax, the body of the match clause may contain any Racket code whatsoever.

A \texttt{match} form may contain several clauses, as in the following function \texttt{leaf?} that recognizes when an $R_0$ node is a leaf. The \texttt{match} proceeds
through the clauses in order, checking whether the pattern can match the input S-expression. The body of the first clause that matches is executed. The output of \texttt{leaf?} for several S-expressions is shown on the right. In the below \texttt{match}, we see another form of pattern: the \((? \texttt{fixnum?})\) applies the predicate \texttt{fixnum?} to the input S-expression to see if it is a machine-representable integer.

\begin{verbatim}
(define (leaf? arith)
  (match arith
    [(? fixnum?) #t]
    [('(read) #t]
    [('- ,c1) #f]
    [('+ ,c1 ,c2) #f])

(leaf? '(read))  #t
(leaf? '(- 8))    #f
(leaf? '(+ (read) (- 8)))  #f
\end{verbatim}

1.4 Recursion

Programs are inherently recursive in that an \(R_0\) expression \((\text{exp})\) is made up of smaller expressions. Thus, the natural way to process an entire program is with a recursive function. As a first example of such a function, we define \texttt{exp?} below, which takes an arbitrary S-expression, \texttt{sexp}, and determines whether or not \texttt{sexp} is an \(R_0\) expression. Note that each match clause corresponds to one grammar rule the body of each clause makes a recursive call for each child node. This pattern of recursive function is so common that it has a name, \textit{structural recursion}. In general, when a recursive function is defined using a sequence of match clauses that correspond to a grammar, and each clause body makes a recursive call on each child node, then we say the function is defined by structural recursion. Below we also define a second function, named \texttt{R0?}, determines whether an S-expression is an \(R_0\) program.
1.4. RECURSION

Indeed, the structural recursion follows the grammar itself. We can generally expect to write a recursive function to handle each non-terminal in the grammar. You may be tempted to write the program with just one function, like this:

```scheme
(define (R0? sexp)
  (match sexp
    [([? fixnum?]) #t]
    ['(read) #t]
    ['(- ,e) (R0? e)]
    ['(+ ,e1 ,e2)
     (and (R0? e1) (R0? e2))]
    [else #f]])
```

Sometimes such a trick will save a few lines of code, especially when it comes to the `program` wrapper. Yet this style is generally not recommended because it can get you into trouble. For instance, the above function is subtly wrong: `(R0? '(program (program 3)))` will return true, when it should return false.

---

1. This principle of structuring code according to the data definition is advocated in the book *How to Design Programs* [http://www.ccs.neu.edu/home/matthias/HtDP2e/](http://www.ccs.neu.edu/home/matthias/HtDP2e/)
1.5 Interpreters

The meaning, or semantics, of a program is typically defined in the specification of the language. For example, the Scheme language is defined in the report by Sperber et al. [2009]. The Racket language is defined in its reference manual [Flatt and PLT, 2014]. In this book we use an interpreter to define the meaning of each language that we consider, following Reynolds’ advice in this regard [Reynolds, 1972]. Here we warm up by writing an interpreter for the $R_0$ language, which serves as a second example of structural recursion. The $\text{interp-R0}$ function is defined in Figure 1.2. The body of the function is a match on the input program $p$ and then a call to the $\text{interp-exp}$ helper function, which in turn has one match clause per grammar rule for $R_0$ expressions.

Let us consider the result of interpreting a few $R_0$ programs. The following program simply adds two integers.

$$(+ 10 32)$$

The result is 42, as you might have expected. Here we have written the program in concrete syntax, whereas the parsed abstract syntax would be the slightly different: $(\text{program } (+ 10 32))$.

The next example demonstrates that expressions may be nested within each other, in this case nesting several additions and negations.

$$(+ 10 (- (+ 12 20)))$$
What is the result of the above program?

As mentioned previously, the \( R_0 \) language does not support arbitrarily-large integers, but only 63-bit integers, so we interpret the arithmetic operations of \( R_0 \) using fixnum arithmetic. What happens when we run the following program?

\[
\text{(define large 999999999999999999)} \\
\text{(interp-R0 '(program (+ (+ (+ large large) (+ large large)) (+ (+ large large) (+ large large))))}
\]

It produces an error:

\text{fx+: result is not a fixnum}

We shall use the convention that if the interpreter for a language produces an error when run on a program, then the meaning of the program is unspecified. The compiler for the language is under no obligation for such a program; it can produce an executable that does anything.

Moving on, the \textbf{read} operation prompts the user of the program for an integer. If we interpret the AST (1.1) and give it the input 50

\[
\text{(interp-R0 ast1.1)}
\]

we get the answer to life, the universe, and everything:

42

We include the \textbf{read} operation in \( R_0 \) so a clever student cannot implement a compiler for \( R_0 \) simply by running the interpreter at compilation time to obtain the output and then generating the trivial code to return the output. (A clever student did this in a previous version of the course.)

The job of a compiler is to translate a program in one language into a program in another language so that the output program behaves the same way as the input program. This idea is depicted in the following diagram.

Suppose we have two languages, \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \), and an interpreter for each language. Suppose that the compiler translates program \( P_1 \) in language \( \mathcal{L}_1 \) into program \( P_2 \) in language \( \mathcal{L}_2 \). Then interpreting \( P_1 \) and \( P_2 \) on their respective interpreters with input \( i \) should yield the same output \( o \).

\[
P_1 \xrightarrow{\text{compile}} P_2 \\
\text{interp-} \mathcal{L}_1(i) \xrightarrow{\text{interp-} \mathcal{L}_2(i)} o
\]
In the next section we see our first example of a compiler, which is another example of structural recursion.

1.6 Example Compiler: a Partial Evaluator

In this section we consider a compiler that translates $R_0$ programs into $R_0$ programs that are more efficient, that is, this compiler is an optimizer. Our optimizer will accomplish this by trying to eagerly compute the parts of the program that do not depend on any inputs. For example, given the following program

$$ (+ (read) (- (+ 5 3))) $$

our compiler will translate it into the program

$$ (+ (read) -8) $$

Figure 1.3 gives the code for a simple partial evaluator for the $R_0$ language. The output of the partial evaluator is an $R_0$ program, which we build up using a combination of quasiquotes and commas. (Though no quasiquote is necessary for integers.) In Figure 1.3, the normal structural recursion is captured in the main `pe-arith` function whereas the code for partially evaluating negation and addition is factored into two separate helper functions: `pe-neg` and `pe-add`. The input to these helper functions is the output of partially evaluating the children nodes.

Our code for `pe-neg` and `pe-add` implements the simple idea of checking whether their arguments are integers and if they are, to go ahead and perform the arithmetic. Otherwise, we use quasiquote to create an AST node for the appropriate operation (either negation or addition) and use comma to splice in the child nodes.

To gain some confidence that the partial evaluator is correct, we can test whether it produces programs that get the same result as the input program. That is, we can test whether it satisfies Diagram (1.7). The following code runs the partial evaluator on several examples and tests the output program. The `assert` function is defined in Appendix 12.2.

```scheme
(define (test-pe p)
  (assert "testing_pe-arith"
    (equal? (interp-R0 p) (interp-R0 (pe-arith p)))))

(test-pe '(+ (read) (- (+ 5 3))))
(test-pe '(+ 1 (+ (read) 1)))
(test-pe '(- (+ (read) (- 5))))
```
1.6. EXAMPLE COMPILER: A PARTIAL EVALUATOR

(define (pe-neg r)
  (cond [(fixnum? r) (fx- 0 r)]
        [else ‘(- ,r)]))

(define (pe-add r1 r2)
  (cond [(and (fixnum? r1) (fixnum? r2)) (fx+ r1 r2)]
        [else ‘(+ ,r1 ,r2)]))

(define (pe-arith e)
  (match e
    [(? fixnum?) e]
    ['(read) ‘(read)]
    ['(- ,e1)
     (pe-neg (pe-arith e1))]
    ['(+ ,e1 ,e2)
     (pe-add (pe-arith e1) (pe-arith e2))])))

Figure 1.3: A partial evaluator for $R_0$ expressions.

Exercise 1. We challenge the reader to improve on the simple partial evaluator in Figure 1.3 by replacing the pe-neg and pe-add helper functions with functions that know more about arithmetic. For example, your partial evaluator should translate

$$(+ 1 (+ (read) 1))$$

into

$$(+ 2 (read))$$

To accomplish this, we recommend that your partial evaluator produce output that takes the form of the residual non-terminal in the following grammar.

\[
\begin{align*}
  exp & ::= \text{int} \mid \text{read} \mid (- \text{read}) \mid (+ \text{exp} \text{exp}) \\
  \text{residual} & ::= \text{int} \mid (+ \text{int} \text{exp}) \mid \text{exp}
\end{align*}
\]
This chapter concerns the challenge of compiling a subset of Racket that includes integer arithmetic and local variable binding, which we name $R_1$, to x86-64 assembly code [Intel 2015]. Henceforth we shall refer to x86-64 simply as x86. The chapter begins with a description of the $R_1$ language (Section 2.1) followed by a description of x86 (Section 2.2). The x86 assembly language is quite large, so we only discuss what is needed for compiling $R_1$. We introduce more of x86 in later chapters. Once we have introduced $R_1$ and x86, we reflect on their differences and come up with a plan to break down the translation from $R_1$ to x86 into a handful of steps (Section 2.3). The rest of the sections in this Chapter give detailed hints regarding each step (Sections 2.4 through 2.10). We hope to give enough hints that the well-prepared reader can implement a compiler from $R_1$ to x86 while at the same time leaving room for some fun and creativity.

2.1 The $R_1$ Language

The $R_1$ language extends the $R_0$ language (Figure 1.1) with variable definitions. The syntax of the $R_1$ language is defined by the grammar in Figure 2.1. The non-terminal `var` may be any Racket identifier. As in $R_0$, `read` is a nullary operator, `-` is a unary operator, and `+` is a binary operator. Similar to $R_0$, the $R_1$ language includes the `program` construct to mark the top of the program, which is helpful in parts of the compiler. The `info` field of the `program` construct contains an association list that is used to communicate auxiliary data from one step of the compiler to the next.

The $R_1$ language is rich enough to exhibit several compilation techniques but simple enough so that the reader, together with couple friends, can
implement a compiler for it in a week or two of part-time work. To give the reader a feeling for the scale of this first compiler, the instructor solution for the \( R_1 \) compiler is less than 500 lines of code.

Let us dive into the description of the \( R_1 \) language. The \texttt{let} construct defines a variable for use within its body and initializes the variable with the value of an expression. So the following program initializes \( x \) to \( 32 \) and then evaluates the body \((+ 10 x)\), producing \( 42 \).

\[
\text{(program () }
\text{(let ([x (+ 12 20)]) (+ 10 x)))}
\]

When there are multiple \texttt{let}'s for the same variable, the closest enclosing \texttt{let} is used. That is, variable definitions overshadow prior definitions. Consider the following program with two \texttt{let}'s that define variables named \( x \).

Can you figure out the result?

\[
\text{(program () }
\text{(let ([x 32]) (+ (let ([x 10]) x) x)))}
\]

For the purposes of showing which variable uses correspond to which definitions, the following shows the \( x \)'s annotated with subscripts to distinguish them. Double check that your answer for the above is the same as your answer for this annotated version of the program.

\[
\text{(program () }
\text{(let ([x_1 32]) (+ (let ([x_2 10]) x_2) x_1)))}
\]

The initializing expression is always evaluated before the body of the \texttt{let}, so in the following, the \texttt{read} for \( x \) is performed before the \texttt{read} for \( y \). Given the input 52 then 10, the following produces 42 (and not -42).

\[
\text{(program () }
\text{(let ([x (read)]) (let ([y (read)]) (+ x (- y))))}
\]

Figure 2.2 shows the interpreter for the \( R_1 \) language. It extends the interpreter for \( R_0 \) with two new \texttt{match} clauses for variables and for \texttt{let}. For \texttt{let}, we will need a way to communicate the initializing value of a variable to all the uses of a variable. To accomplish this, we maintain a mapping from variables to values, which is traditionally called an \textit{environment}. For

\[
\begin{align*}
\text{exp} & ::= \text{int} | (\text{read}) | (- \text{exp}) | (+ \text{exp} \text{exp}) \\
& \quad | \text{var} | (\text{let} ([\text{var} \text{exp}]) \text{exp}) \\
\text{R}_1 & ::= \text{(program info exp)}
\end{align*}
\]
2.1. THE $R_1$ LANGUAGE

(define (interp-exp env)
  (lambda (e)
    (match e
      [(? fixnum?) e]
      ['(read)
        (define r (read))
        (cond [[(fixnum? r) r]
                  [else (error 'interp-R1 "expected an integer" r)])]
      ['(- ,e)
        (define v ((interp-exp env) e))
        (fx- 0 v)]
      ['(+ ,e1 ,e2)
        (define v1 ((interp-exp env) e1))
        (define v2 ((interp-exp env) e2))
        (fx+ v1 v2)]
      [(? symbol?) (lookup e env)]
      ['(let (,x ,e) ,body)
        (define new-env (cons (cons x ((interp-exp env) e)) env))
        ((interp-exp new-env) body)])
    ))

(define (interp-R1 env)
  (lambda (p)
    (match p
      [('(program ,info ,e) ((interp-exp '()) e]])

Figure 2.2: Interpreter for the $R_1$ language.

simplicity, here we use an association list to represent the environment. The \texttt{interp-R1} function takes the current environment, \texttt{env}, as an extra parameter. When the interpreter encounters a variable, it finds the corresponding value using the \texttt{lookup} function (Appendix 12.2). When the interpreter encounters a \texttt{let}, it evaluates the initializing expression, extends the environment with the result bound to the variable, then evaluates the body of the \texttt{let}.

The goal for this chapter is to implement a compiler that translates any program $P_1$ in the $R_1$ language into an x86 assembly program $P_2$ such that $P_2$ exhibits the same behavior on an x86 computer as the $R_1$ program running in a Racket implementation. That is, they both output the same
2. INTEGERS AND VARIABLES

integer $n$.

\[ P_1 \xrightarrow{\text{compile}} P_2 \xrightarrow{\text{interp-R1}} \xrightarrow{\text{interp-x86}} n \]

In the next section we introduce enough of the x86 assembly language to compile $R_1$.

2.2 The x86 Assembly Language

An x86 program is a sequence of instructions. The program is stored in the computer’s memory and the program counter points to the address of the next instruction to be executed. For most instructions, once the instruction is executed, the program counter is incremented to point to the immediately following instruction in memory. Each instruction may refer to integer constants (called immediate values), variables called registers, and instructions may load and store values into memory. For our purposes, we can think of the computer’s memory as a mapping of 64-bit addresses to 64-bit values\(^1\). Figure 2.3 defines the syntax for the subset of the x86 assembly language needed for this chapter. We use the AT&T syntax expected by the GNU assembler, which comes with the gcc compiler that we use for compiling assembly code to machine code. Also, Appendix [12.3] includes a quick-reference of all the x86 instructions used in this book and a short explanation of what they do.

An immediate value is written using the notation \$n where $n$ is an integer. A register is written with a % followed by the register name, such as %rax. An access to memory is specified using the syntax $n(%r)$, which obtains the address stored in register $r$ and then offsets the address by $n$ bytes (8 bits). The address is then used to either load or store to memory depending on whether it occurs as a source or destination argument of an instruction.

An arithmetic instruction, such as \texttt{addq} $s, d$, reads from the source $s$ and destination $d$, applies the arithmetic operation, then writes the result in $d$.

---

\(^1\)This simple story suffices for describing how sequential programs access memory but is not sufficient for multi-threaded programs. However, multi-threaded execution is beyond the scope of this book.
2.2. THE X86 ASSEMBLY LANGUAGE

\[
\text{reg} ::= \text{rsp} | \text{rbp} | \text{rax} | \text{rbx} | \text{rcx} | \text{rdx} | \text{rsi} | \text{rdi} | \\
\text{r8} | \text{r9} | \text{r10} | \text{r11} | \text{r12} | \text{r13} | \text{r14} | \text{r15}
\]

\[
\text{arg} ::= \$ \text{int} | \%\text{reg} | \text{int}(\%\text{reg})
\]

\[
\text{instr} ::= \text{addq} \text{arg}, \text{arg} | \text{subq} \text{arg}, \text{arg} | \text{negq} \text{arg} | \text{movq} \text{arg}, \text{arg} | \\
\text{callq label} | \text{pushq arg} | \text{popq arg} | \text{retq} | \text{label: instr}
\]

\[
\text{prog} ::= \text{.globl main} \\
\text{main: instr}^+
\]

Figure 2.3: A subset of the x86 assembly language (AT&T syntax).

\[
\text{.globl main} \\
\text{main:} \\
\text{movq } \$10, \%\text{rax} \\
\text{addq } \$32, \%\text{rax} \\
\text{retq}
\]

Figure 2.4: An x86 program equivalent to (+ 10 32).

The move instruction, \text{movq s d} reads from \text{s} and stores the result in \text{d}. The \text{callq label} instruction executes the procedure specified by the label.

Figure 2.4 depicts an x86 program that is equivalent to (+ 10 32). The \text{globl} directive says that the \text{main} procedure is externally visible, which is necessary so that the operating system can call it. The label \text{main:} indicates the beginning of the \text{main} procedure which is where the operating system starts executing this program. The instruction \text{movq } \$10, \%\text{rax} puts 10 into register \text{rax}. The following instruction \text{addq } \$32, \%\text{rax} adds 32 to the 10 in \text{rax} and puts the result, 42, back into \text{rax}.

The last instruction, \text{retq}, finishes the \text{main} function by returning the integer in \text{rax} to the operating system. The operating system interprets this integer as the program’s exit code. By convention, an exit code of 0 indicates the program was successful, and all other exit codes indicate various errors. Nevertheless, we return the result of the program as the exit code.

Unfortunately, x86 varies in a couple ways depending on what operating system it is assembled in. The code examples shown here are correct on Linux and most Unix-like platforms, but when assembled on Mac OS X, labels like \text{main} must be prefixed with an underscore, as in _\text{main}.

We exhibit the use of memory for storing intermediate results in the next example. Figure 2.5 lists an x86 program that is equivalent to (+ 52 (- 10)).
This program uses a region of memory called the \textit{procedure call stack} (or \textit{stack} for short). The stack consists of a separate \textit{frame} for each procedure call. The memory layout for an individual frame is shown in Figure \ref{fig:stack_layout}. The register \texttt{rsp} is called the \textit{stack pointer} and points to the item at the top of the stack. The stack grows downward in memory, so we increase the size of the stack by subtracting from the stack pointer. The frame size is required to be a multiple of 16 bytes. In the context of a procedure call, the \textit{return address} is the next instruction on the caller side that comes after the call instruction. During a function call, the return address is pushed onto the stack. The register \texttt{rbp} is the \textit{base pointer} which serves two purposes: 1) it saves the location of the stack pointer for the calling procedure and 2) it is used to access variables associated with the current procedure. The base pointer of the calling procedure is pushed onto the stack after the return address. We number the variables from 1 to \(n\). Variable 1 is stored at address \(-8(\%\text{rbp})\), variable 2 at \(-16(\%\text{rbp})\), etc.

Getting back to the program in Figure \ref{fig:x86_example}, the first three instructions are the typical \textit{prelude} for a procedure. The instruction \texttt{pushq \%\text{rbp}} saves the base pointer for the procedure that called the current one onto the stack and subtracts 8 from the stack pointer. The second instruction \texttt{movq \%\text{rsp}, \%\text{rbp}} changes the base pointer to the top of the stack. The instruction \texttt{subq}
$16$, %rsp moves the stack pointer down to make enough room for storing variables. This program just needs one variable (8 bytes) but because the frame size is required to be a multiple of 16 bytes, it rounds to 16 bytes.

The four instructions under the label start carry out the work of computing (+ 52 (- 10)). The first instruction `movq $10, -8(%rbp)` stores 10 in variable 1. The instruction `negq -8(%rbp)` changes variable 1 to −10. The `movq $52, %rax` places 52 in the register %rax and `addq -8(%rbp), %rax` adds the contents of variable 1 to %rax, at which point %rax contains 42.

The three instructions under the label conclusion are the typical finale of a procedure. The first two are necessary to get the state of the machine back to where it was at the beginning of the procedure. The `addq $16, %rsp` instruction moves the stack pointer back to point at the old base pointer. The amount added here needs to match the amount that was subtracted in the prelude of the procedure. Then `popq %rbp` returns the old base pointer to %rbp and adds 8 to the stack pointer. The last instruction, `retq`, jumps back to the procedure that called this one and adds 8 to the stack pointer, which returns the stack pointer to where it was prior to the procedure call.

The compiler will need a convenient representation for manipulating x86 programs, so we define an abstract syntax for x86 in Figure 2.7. We refer to this language as x860 with a subscript 0 because later we introduce extended versions of this assembly language. The main difference compared to the concrete syntax of x86 (Figure 2.3) is that it does not allow labelled instructions to appear anywhere, but instead organizes instructions into groups called blocks and a label is associated with every block, which is why the program form includes an association list mapping labels to blocks. The reason for this organization becomes apparent in Chapter 4.
2.3 Planning the trip to x86 via the $C_0$ language

To compile one language to another it helps to focus on the differences between the two languages because the compiler will need to bridge them. What are the differences between $R_1$ and x86 assembly? Here we list some of the most important ones.

(a) x86 arithmetic instructions typically have two arguments and update the second argument in place. In contrast, $R_1$ arithmetic operations take two arguments and produce a new value. An x86 instruction may have at most one memory-accessing argument. Furthermore, some instructions place special restrictions on their arguments.

(b) An argument to an $R_1$ operator can be any expression, whereas x86 instructions restrict their arguments to be simple expressions like integers, registers, and memory locations. (All the other kinds are called complex expressions.)

(c) The order of execution in x86 is explicit in the syntax: a sequence of instructions and jumps to labeled positions, whereas in $R_1$ it is a left-to-right depth-first traversal of the abstract syntax tree.

(d) An $R_1$ program can have any number of variables whereas x86 has 16 registers and the procedure calls stack.

(e) Variables in $R_1$ can overshadow other variables with the same name. The registers and memory locations of x86 all have unique names or addresses.

We ease the challenge of compiling from $R_1$ to x86 by breaking down the problem into several steps, dealing with the above differences one at a time.
Each of these steps is called a pass of the compiler, because step traverses (passes over) the AST of the program. We begin by giving a sketch about how we might implement each pass, and give them names. We shall then figure out an ordering of the passes and the input/output language for each pass. The very first pass has $R_1$ as its input language and the last pass has x86 as its output language. In between we can choose whichever language is most convenient for expressing the output of each pass, whether that be $R_1$, x86, or new intermediate languages of our own design. Finally, to implement the compiler, we shall write one function, typically a structural recursive function, per pass.

**Pass select-instructions** To handle the difference between $R_1$ operations and x86 instructions we shall convert each $R_1$ operation to a short sequence of instructions that accomplishes the same task.

**Pass remove-complex-opera** To ensure that each subexpression (i.e. operator and operand, and hence opera*) is a simple expression, we shall introduce temporary variables to hold the results of subexpressions.

**Pass explicate-control** To make the execution order of the program explicit, we shall convert from the abstract syntax tree representation into a graph representation in which each node contains a sequence of actions and the edges say where to go after the sequence is complete.

**Pass assign-homes** To handle the difference between the variables in $R_1$ versus the registers and stack location in x86, we shall come up with an assignment of each variable to its home, that is, to a register or stack location.

**Pass uniquify** This pass deals with the shadowing of variables by renaming every variable to a unique name, so that shadowing no longer occurs.

The next question is: in what order should we apply these passes? This question can be a challenging one to answer because it is difficult to know ahead of time which orders will be better (easier to implement, produce more efficient code, etc.) so often some trial-and-error is involved. Nevertheless, we can try to plan ahead and make educated choices regarding the orderings.

Let us consider the ordering of uniquify and remove-complex-opera*. The assignment of subexpressions to temporary variables involves introducing new variables and moving subexpressions, which might change the shadowing of variables and inadvertently change the behavior of the program. But if we apply uniquify first, this will not be an issue. Of course, this
means that in remove-complex-opera*, we need to ensure that the temporary variables that it creates are unique.

Next we shall consider the ordering of the explicate-control pass and select-instructions. It is clear that explicate-control must come first because the control-flow graph that it generates is needed when determining where to place the x86 label and jump instructions. Regarding the ordering of explicate-control with respect to uniquify, it is important to apply uniquify first because in explicate-control we change all the let-bound variables to become local variables whose scope is the entire program. With respect to remove-complex-opera*, it perhaps does not matter very much, but it works well to place explicate-control after removing complex subexpressions.

The assign-homes pass should come after remove-complex-opera* and explicate-control. The remove-complex-opera* pass generates temporary variables, which also need to be assigned homes. The explicate-control pass deletes branches that will never be executed, which can remove variables. Thus it is good to place explicate-control prior to assign-homes so that there are fewer variables that need to be assigned homes. This is important because the assign-homes pass has the highest time complexity.

Last, we need to decide on the ordering of select-instructions and assign-homes. These two issues are intertwined, creating a bit of a Gordian Knot. To do a good job of assigning homes, it is helpful to have already determined which instructions will be used, because x86 instructions have restrictions about which of their arguments can be registers versus stack locations. For example, one can give preferential treatment to variables that occur in register-argument positions. On the other hand, it may turn out to be impossible to make sure that all such variables are assigned to registers, and then one must redo the selection of instructions. Some compilers handle this problem by iteratively repeating these two passes until a good solution is found. We shall use a simpler approach in which select-instructions comes first, followed by the assign-homes, followed by a third pass, named patch-instructions, that uses a reserved register (rax) to patch-up outstanding problems regarding instructions with too many memory accesses.

Figure 2.8 presents the ordering of the compiler passes in the form of a graph. Each pass is an edge and the input/output language of each pass is a node in the graph. The output of uniquify and remove-complex-opera* are programs that are still in the $R_1$ language, but the output of the pass explicate-control is in a different language that is designed to make the order of evaluation explicit in its syntax, which we introduce in the next section. Also, there are two passes of lesser importance in Figure 2.8 that
2.3. PLANNING THE TRIP TO X86 VIA THE $C_0$ LANGUAGE

![Diagram of passes for compiling $R_1$.](image)

Figure 2.8: Overview of the passes for compiling $R_1$.

we have not yet talked about, $uncover$-$locals$ and $print$-$x86$. We shall discuss them later in this Chapter.

### 2.3.1 The $C_0$ Intermediate Language

It so happens that the output of $explicate$-$control$ is vaguely similar to the $C$ language [Kernighan and Ritchie, 1988], so we name it $C_0$. The syntax for $C_0$ is defined in Figure 2.9. The $C_0$ language supports the same operators as $R_1$ but the arguments of operators are now restricted to just variables and integers, thanks to the $remove$-$complex$-$operators$ pass. In the literature this style of intermediate language is called administrative normal form, or ANF for short [Danvy, 1991, Flanagan et al., 1993]. Instead of $let$ expressions, $C_0$ has assignment statements which can be executed in sequence using the $seq$ construct. A sequence of statements always ends with $return$, a guarantee that is baked into the grammar rules for the $tail$ non-terminal. The naming of this non-terminal comes from the term $tail$ position, which refers to an expression that is the last one to execute within a function. (A expression in tail position may contain subexpressions, and those may or may not be in tail position depending on the kind of expression.)

A $C_0$ program consists of an association list mapping labels to tails. This is overkill for the present Chapter, as we do not yet need to introduce $goto$ for jumping to labels, but it saves us from having to change the syntax of the program construct in Chapter 4. For now there will be just one label, $start$, and the whole program will be its tail. The $info$ field of the program construct, after the $uncover$-$locals$ pass, will contain a mapping from the symbol $locals$ to a list of variables, that is, a list of all the variables used in
2. INTEGERS AND VARIABLES

\[
\begin{align*}
\text{arg} & ::= \text{int} \mid \text{var} \\
\text{exp} & ::= \text{arg} \mid (\text{read}) \mid (- \text{arg}) \mid (+ \text{arg} \text{arg}) \\
\text{stmt} & ::= (\text{assign} \ \text{var} \ \text{exp}) \\
\text{tail} & ::= (\text{return} \ \text{exp}) \mid (\text{seq} \ \text{stmt} \ \text{tail}) \\
\text{C}_0 & ::= (\text{program} \ \text{info} \ (\text{label} \ . \ \text{tail})^+))
\end{align*}
\]

Figure 2.9: The \( \text{C}_0 \) intermediate language.

the program. At the start of the program, these variables are uninitialized (they contain garbage) and each variable becomes initialized on its first assignment.

2.3.2 The dialects of x86

The x86\( ^* \) language, pronounced “pseudo-x86”, is the output of the pass select-instructions. It extends x86\( _0 \) with variables and looser rules regarding instruction arguments. The x86\( ^1 \) language, the output of print-x86, is the concrete syntax for x86.

2.4 Uniquify Variables

The purpose of this pass is to make sure that each let uses a unique variable name. For example, the uniquify pass should translate the program on the left into the program on the right.

\[
\begin{align*}
\text{(program \ ()} \\
\text{(let \ ([x 32])} \\
\text{(+ \ (let \ ([x 10]) \ x)))} & \Rightarrow \\
\text{(program \ ()} \\
\text{(let \ ([x.1 32])} \\
\text{(+ \ (let \ ([x.2 10]) \ x.2) \ x.1)))}
\end{align*}
\]

The following is another example translation, this time of a program with a let nested inside the initializing expression of another let.

\[
\begin{align*}
\text{(program \ ()} \\
\text{(let \ ([x \ (let \ ([x 4])} \\
\text{(+ x 1))))} & \Rightarrow \\
\text{(program \ ()} \\
\text{(let \ ([x.2 \ (let \ ([x.1 4])} \\
\text{(+ x.1 1))))} \\
\text{(+ x.2 2))})
\end{align*}
\]

We recommend implementing uniquify as a structurally recursive function that mostly copies the input program. However, when encountering a let, it should generate a unique name for the variable (the Racket function gensym is handy for this) and associate the old name with the new unique name in an association list. The uniquify function will need to access this
2.4. UNIQUIFY VARIABLES

```
(define (uniquify-exp alist)
  (lambda (e)
    (match e
      [((? symbol?) ___]
      [((? integer?) e]
      [let ([,x ,e]) ,body) ___]
      ['(op ,es ...)
        '(op ,@(for/list ([e es]) ((uniquify-exp alist) e)))])))

(define (uniquify alist)
  (lambda (e)
    (match e
      [program ,info ,e)
        '(program ,info ,((uniquify-exp alist) e))])))
```

Figure 2.10: Skeleton for the uniquify pass.

association list when it gets to a variable reference, so we add another parameter to uniquify for the association list. It is quite common for a compiler pass to need a map to store extra information about variables. Such maps are often called symbol tables.

The skeleton of the uniquify function is shown in Figure 2.10. The function is curried so that it is convenient to partially apply it to an association list and then apply it to different expressions, as in the last clause for primitive operations in Figure 2.10. In the last match clause for the primitive operators, note the use of the comma-@ operator to splice a list of S-expressions into an enclosing S-expression.

Exercise 2. Complete the uniquify pass by filling in the blanks, that is, implement the clauses for variables and for the let construct.

Exercise 3. Test your uniquify pass by creating five example R₁ programs and checking whether the output programs produce the same result as the input programs. The R₁ programs should be designed to test the most interesting parts of the uniquify pass, that is, the programs should include let constructs, variables, and variables that overshadow each other. The five programs should be in a subdirectory named tests and they should have the same file name except for a different integer at the end of the name, followed
by the ending .rkt. Use the interp-tests function (Appendix [12.2] from utilities.rkt to test your uniquify pass on the example programs.

### 2.5 Remove Complex Operators and Operands

The `remove-complex-opera*` pass will transform $R_1$ programs so that the arguments of operations are simple expressions. Put another way, this pass removes complex subexpressions, such as the expression $(- 10)$ in the program below. This is accomplished by introducing a new `let`-bound variable, binding the complex subexpression to the new variable, and then using the new variable in place of the complex expression, as shown in the output of `remove-complex-opera*` on the right.

```
(program ()
 (+ 52 (- 10)))
⇒
(let ([tmp.1 (- 10)])
  (+ 52 tmp.1))
```

We recommend implementing this pass with two mutually recursive functions, `rco-arg` and `rco-exp`. The idea is to apply `rco-arg` to subexpressions that need to become simple and to apply `rco-exp` to subexpressions can stay complex. Both functions take an expression in $R_1$ as input. The `rco-exp` function returns an expression. The `rco-arg` function returns two things: a simple expression and association list mapping temporary variables to complex subexpressions. You can return multiple things from a function using Racket’s `values` form and you can receive multiple things from a function call using the `define-values` form. If you are not familiar with these constructs, the Racket documentation will be of help. Also, the `for/lists` construct is useful for applying a function to each element of a list, in the case where the function returns multiple values.

```
(rco-arg ‘(- 10))
⇒
(values ‘tmp.1
  ‘((tmp.1 . (- 10))))
```

Take special care of programs such as the following that `let`-bind variables with integers or other variables. It should leave them unchanged, as shown in to the program on the right

```
(program ()
  (let ([a 42])
    (let ([b a]
      b)))
⇒
(let ([a 42])
  (let ([b a]
    b)))
```

and not translate them to the following, which might result from a careless
2.6. **EXPlicate CONTROL**

Implementation of `rco-exp` and `rco-args`.

```racket
(program ()
  (let ([tmp.1 42])
    (let ([a tmp.1])
      (let ([tmp.2 a])
        (let ([b tmp.2])
          b))))
```

**Exercise 4.** Implement the `remove-complex-opera*` pass and test it on all of the example programs that you created to test the `uniquify` pass and create three new example programs that are designed to exercise all of the interesting code in the `remove-complex-opera*` pass. Use the `interp-tests` function (Appendix 12.2) from `utilities.rkt` to test your passes on the example programs.

### 2.6 Explicate Control

The **explicate-control** pass makes the order of execution explicit in the syntax of the program. For $R_1$, this amounts to flattening `let` constructs into a sequence of assignment statements. For example, consider the following $R_1$ program.

```racket
(program ()
  (let ([y (let ([x 20])
               (+ x (let ([x 22]) x)))]
        y))
```

The output of `remove-complex-opera*` is shown below, on the left. The right-hand-side of a `let` executes before its body, so the order of evaluation for this program is to assign 20 to `x.1`, assign 22 to `x.2`, assign `(+ x.1 x.2)` to `y`, then return `y`. Indeed, the result of `explicate-control` produces code in the $C_0$ language that makes this explicit.

```racket
(program ()
  (let ([y (let ([x.1 20])
              (let ([x.2 22])
                (+ x.1 x.2)])]
        y))
```

```racket
(program ()
  ((start .
    (seq (assign x.1 20)
         (seq (assign x.2 22)
              (seq (assign y (+ x.1 x.2))
                   (return y))))))
```

We recommend implementing `explicate-control` using two mutually recursive functions: `explicate-control-tail` and `explicate-control-assign`. The `explicate-control-tail` function should be applied to expressions
in tail position, whereas \texttt{explicate-control-assign} should be applied
to expressions that occur on the right-hand-side of a \texttt{let}. The function
\texttt{explicate-control-tail} takes an $R_1$ expression as input and produces a
$C_0$ tail (see the grammar in Figure 2.9). The \texttt{explicate-control-assign}
function takes an $R_1$ expression, the variable that it is to be assigned to,
and $C_0$ code (a tail) that should come after the assignment (e.g., the code
generated for the body of the let).

\section{Uncover Locals}

The pass \texttt{uncover-locals} simply collects all of the variables in the program
and places then in the \texttt{info} of the program construct. Here is the output for
the example program of the last section.

\begin{verbatim}
(program ((locals . (x.1 x.2 y)))
  ((start .
    (seq (assign x.1 20)
    (seq (assign x.2 22)
    (seq (assign y (+ x.1 x.2))
    (return y))))))))
\end{verbatim}

\section{Select Instructions}

In the \texttt{select-instructions} pass we begin the work of translating from
$C_0$ to x86. The target language of this pass is a pseudo-x86 language that
still uses variables, so we add an AST node of the form (\texttt{var var}) to the x86
abstract syntax. We recommend implementing the \texttt{select-instructions}
in terms of three auxiliary functions, one for each of the non-terminals of
$C_0$: \texttt{arg}, \texttt{stmt}, and \texttt{tail}.

The cases for \texttt{arg} are straightforward, simply putting variables and in-
teger literals into the s-expression format expected of pseudo-x86, (\texttt{var x})
and (\texttt{int n}), respectively.

Next we discuss some of the cases for \texttt{stmt}, starting with arithmetic op-
erations. For example, in $C_0$ an addition operation can take the form below.
To translate to x86, we need to use the \texttt{addq} instruction which does an in-
place update. So we must first move 10 to x.

\begin{verbatim}
(assign x (+ 10 32)) ⇒ (movq (int 10) (var x)) (addq (int 32) (var x))
\end{verbatim}

There are some cases that require special care to avoid generating needlessly
complicated code. If one of the arguments is the same as the left-hand side of the assignment, then there is no need for the extra move instruction. For example, the following assignment statement can be translated into a single addq instruction.

\[(\text{assign } x \ (+ \ 10 \ x)) \Rightarrow (\text{addq} \ (\text{int} \ 10) \ (\text{var} \ x))\]

The \text{read} operation does not have a direct counterpart in x86 assembly, so we have instead implemented this functionality in the C language, with the function \text{read_int} in the file \text{runtime.c}. In general, we refer to all of the functionality in this file as the \text{runtime system}, or simply the \text{runtime} for short. When compiling your generated x86 assembly code, you will need to compile \text{runtime.c} to \text{runtime.o} (an “object file”, using \text{gcc} option \text{-c}) and link it into the final executable. For our purposes of code generation, all you need to do is translate an assignment of \text{read} to some variable \text{lhs} (for left-hand side) into a call to the \text{read_int} function followed by a move from \text{rax} to the left-hand side. The move from \text{rax} is needed because the return value from \text{read_int} goes into \text{rax}, as is the case in general.

\[(\text{assign } \text{lhs} \ (\text{read})) \Rightarrow (\text{callq} \ \text{read_int})\]

\[(\text{movq} \ (\text{reg} \ \text{rax}) \ (\text{var} \ \text{lhs}))\]

There are two cases for the \text{tail} non-terminal: \text{return} and \text{seq}. Regarding (\text{return} e), we recommend treating it as an assignment to the \text{rax} register followed by a jump to the conclusion of the program (so the conclusion needs to be labeled). For (\text{seq} st), we simply process the statement \text{s} and tail \text{t} recursively and append the resulting instructions.

Exercise 5. Implement the \text{select-instructions} pass and test it on all of the example programs that you created for the previous passes and create three new example programs that are designed to exercise all of the interesting code in this pass. Use the \text{interp-tests} function (Appendix 12.2) from \text{utilities.rkt} to test your passes on the example programs.

2.9 Assign Homes

As discussed in Section 2.3, the \text{assign-homes} pass places all of the variables on the stack. Consider again the example \text{R1} program \((+ \ 52 \ (- \ 10))\), which after \text{select-instructions} looks like the following.

\[(\text{movq} \ (\text{int} \ 10) \ (\text{var} \ \text{tmp.1}))\]
\[(\text{negq} \ (\text{var} \ \text{tmp.1}))\]
2. INTEGERS AND VARIABLES

(movq (var tmp.1) (var tmp.2))
(addq (int 52) (var tmp.2))
(movq (var tmp.2) (reg rax)))

The variable tmp.1 is assigned to stack location -8(%rbp), and tmp.2 is assign to -16(%rbp), so the assign-homes pass translates the above to

(movq (int 10) (deref rbp -8))
(negq (deref rbp -8))
(movq (deref rbp -8) (deref rbp -16))
(addq (int 52) (deref rbp -16))
(movq (deref rbp -16) (reg rax)))

In the process of assigning stack locations to variables, it is convenient to compute and store the size of the frame (in bytes) in the info field of the program node, with the key stack-space, which will be needed later to generate the procedure conclusion. Some operating systems place restrictions on the frame size. For example, Mac OS X requires the frame size to be a multiple of 16 bytes.

Exercise 6. Implement the assign-homes pass and test it on all of the example programs that you created for the previous passes pass. We recommend that assign-homes take an extra parameter that is a mapping of variable names to homes (stack locations for now). Use the interp-tests function (Appendix 12.2) from utilities.rkt to test your passes on the example programs.

2.10 Patch Instructions

The purpose of this pass is to make sure that each instruction adheres to the restrictions regarding which arguments can be memory references. For most instructions, the rule is that at most one argument may be a memory reference.

Consider again the following example.

(let ([a 42])
 (let ([b a])
  b))

After assign-homes pass, the above has been translated to

(movq (int 42) (deref rbp -8))
(movq (deref rbp -8) (deref rbp -16))
(movq (deref rbp -16) (reg rax))
(jmp conclusion)
2.11. PRINT X86

The second movq instruction is problematic because both arguments are stack locations. We suggest fixing this problem by moving from the source to the register rax and then from rax to the destination, as follows.

\[
\begin{align*}
&\text{movq (int 42) (deref rbp -8)} \\
&\text{movq (deref rbp -8) (reg rax)} \\
&\text{movq (reg rax) (deref rbp -16)} \\
&\text{movq (deref rbp -16) (reg rax)}
\end{align*}
\]

**Exercise 7.** Implement the patch-instructions pass and test it on all of the example programs that you created for the previous passes and create three new example programs that are designed to exercise all of the interesting code in this pass. Use the interp-tests function (Appendix 12.2) from utilities.rkt to test your passes on the example programs.

2.11 Print x86

The last step of the compiler from R₁ to x86 is to convert the x86 AST (defined in Figure 2.7) to the string representation (defined in Figure 2.3). The Racket format and string-append functions are useful in this regard. The main work that this step needs to perform is to create the main function and the standard instructions for its prelude and conclusion, as shown in Figure 2.5 of Section 2.2. You need to know the number of stack-allocated variables, so we suggest computing it in the assign-homes pass (Section 2.9) and storing it in the info field of the program node.

If you want your program to run on Mac OS X, your code needs to determine whether or not it is running on a Mac, and prefix underscores to labels like main. You can determine the platform with the Racket call (system-type ’os), which returns ’macosx, ’unix, or ’windows.

**Exercise 8.** Implement the print-x86 pass and test it on all of the example programs that you created for the previous passes. Use the compiler-tests function (Appendix 12.2) from utilities.rkt to test your complete compiler on the example programs.
2. INTEGERS AND VARIABLES
Register Allocation

In Chapter 2 we simplified the generation of x86 assembly by placing all variables on the stack. We can improve the performance of the generated code considerably if we instead place as many variables as possible into registers. The CPU can access a register in a single cycle, whereas accessing the stack takes many cycles to go to cache or many more to access main memory. Figure 3.1 shows a program with four variables that serves as a running example. We show the source program and also the output of instruction selection. At that point the program is almost x86 assembly but not quite; it still contains variables instead of stack locations or registers.

The goal of register allocation is to fit as many variables into registers as possible. It is often the case that we have more variables than registers, so we cannot map each variable to a different register. Fortunately, it is common for different variables to be needed during different periods of time, and in such cases several variables can be mapped to the same register. Consider variables $x$ and $y$ in Figure 3.1. After the variable $x$ is moved to $z$ it is no longer needed. Variable $y$, on the other hand, is used only after this point, so $x$ and $y$ could share the same register. The topic of Section 3.2 is how we compute where a variable is needed. Once we have that information, we compute which variables are needed at the same time, i.e., which ones interfere, and represent this relation as graph whose vertices are variables and edges indicate when two variables interfere with each other (Section 3.3). We then model register allocation as a graph coloring problem, which we discuss in Section 3.4.

In the event that we run out of registers despite these efforts, we place the remaining variables on the stack, similar to what we did in Chapter 2. It is common to say that when a variable that is assigned to a stack location,
3. REGISTER ALLOCATION

After instruction selection:

\[
\begin{align*}
R_1 \text{ program:} & \quad \text{(movq (int 1) (var v))} \\
\text{(program ()} & \quad \text{(movq (int 46) (var w))} \text{)} \\
\text{(let ([v 1])} & \quad \text{(movq (var v) (var x))} \text{)} \\
\text{(let ([w 46])} & \quad \text{(addq (int 7) (var x))} \text{)} \\
\text{(let ([x (+ v 7)])} & \quad \text{(movq (var x) (var y))} \text{)} \\
\text{(let ([y (+ 4 x)])} & \quad \text{(addq (int 4) (var y))} \text{)} \\
\text{(let ([z (+ x w)])} & \quad \text{(movq (var x) (var z))} \text{)} \\
\text{(+ z (- y)))))))) & \quad \text{(addq (var w) (var z))} \text{) } \\
& \quad \text{(movq (var y) (var t.1))} \text{) } \\
& \quad \text{(negq (var t.1))} \text{) } \\
& \quad \text{(movq (var z) (reg rax))} \text{) } \\
& \quad \text{(addq (var t.1) (reg rax))} \text{) } \\
& \quad \text{(jmp conclusion)))))} \text{)} \\
\end{align*}
\]

Figure 3.1: An example program for register allocation.

it has been spilled. The process of spilling variables is handled as part of the graph coloring process described in 3.4.

3.1 Registers and Calling Conventions

As we perform register allocation, we will need to be aware of the conventions that govern the way in which registers interact with function calls. The convention for x86 is that the caller is responsible for freeing up some registers, the caller-saved registers, prior to the function call, and the callee is responsible for saving and restoring some other registers, the callee-saved registers, before and after using them. The caller-saved registers are

\[
\begin{align*}
\text{rax, rdx, rcx, rsi, rdi, r8, r9, r10, r11} \\
\end{align*}
\]

while the callee-saved registers are

\[
\begin{align*}
\text{rsp, rbp, rbx, r12, r13, r14, r15} \\
\end{align*}
\]

Another way to think about this caller/callee convention is the following. The caller should assume that all the caller-saved registers get overwritten with arbitrary values by the callee. On the other hand, the caller can safely assume that all the callee-saved registers contain the same values after the
call that they did before the call. The callee can freely use any of the caller-
saved registers. However, if the callee wants to use a callee-saved register, 
the callee must arrange to put the original value back in the register prior to 
returning to the caller, which is usually accomplished by saving and restoring 
the value from the stack.

3.2 Liveness Analysis

A variable is live if the variable is used at some later point in the program 
and there is not an intervening assignment to the variable. To understand 
the latter condition, consider the following code fragment in which there are 
two writes to b. Are a and b both live at the same time?

1. (movq (int 5) (var a))
2. (movq (int 30) (var b))
3. (movq (var a) (var c))
4. (movq (int 10) (var b))
5. (addq (var b) (var c))

The answer is no because the value 30 written to b on line 2 is never used. 
The variable b is read on line 5 and there is an intervening write to b on 
line 4, so the read on line 5 receives the value written on line 4, not line 2. 

The live variables can be computed by traversing the instruction se-
quenue back to front (i.e., backwards in execution order). Let \( I_1, \ldots, I_n \) be 
the instruction sequence. We write \( L_{after}(k) \) for the set of live variables after 
instruction \( I_k \) and \( L_{before}(k) \) for the set of live variables before instruction 
\( I_k \). The live variables after an instruction are always the same as the live 
variables before the next instruction. 

\[
L_{after}(k) = L_{before}(k + 1)
\]

To start things off, there are no live variables after the last instruction, so 

\[
L_{after}(n) = \emptyset
\]

We then apply the following rule repeatedly, traversing the instruction se-
quenue back to front.

\[
L_{before}(k) = (L_{after}(k) - W(k)) \cup R(k),
\]

where \( W(k) \) are the variables written to by instruction \( I_k \) and \( R(k) \) are 
the variables read by instruction \( I_k \). Figure 3.2 shows the results of live 
variables analysis for the running example, with each instruction aligned 
with its \( L_{after} \) to make the figure easy to read.
Exercise 9. Implement the compiler pass named `uncover-live` that computes the live-after sets. We recommend storing the live-after sets (a list of lists of variables) in the `info` field of the `block` construct. We recommend organizing your code to use a helper function that takes a list of instructions and an initial live-after set (typically empty) and returns the list of live-after sets. We recommend creating helper functions to 1) compute the set of variables that appear in an argument (of an instruction), 2) compute the variables read by an instruction which corresponds to the $R$ function discussed above, and 3) the variables written by an instruction which corresponds to $W$.

3.3 Building the Interference Graph

Based on the liveness analysis, we know where each variable is needed. However, during register allocation, we need to answer questions of the specific form: are variables $u$ and $v$ live at the same time? (And therefore cannot be assigned to the same register.) To make this question easier to answer, we create an explicit data structure, an interference graph. An interference graph is an undirected graph that has an edge between two variables if they are live at the same time, that is, if they interfere with each other.

The most obvious way to compute the interference graph is to look at the set of live variables between each statement in the program, and add an edge to the graph for every pair of variables in the same set. This approach...
is less than ideal for two reasons. First, it can be rather expensive because it takes $O(n^2)$ time to look at every pair in a set of $n$ live variables. Second, there is a special case in which two variables that are live at the same time do not actually interfere with each other: when they both contain the same value because we have assigned one to the other.

A better way to compute the interference graph is to focus on the writes. That is, for each instruction, create an edge between the variable being written to and all the other live variables. (One should not create self edges.) For a callq instruction, think of all caller-saved registers as being written to, so and edge must be added between every live variable and every caller-saved register. For movq, we deal with the above-mentioned special case by not adding an edge between a live variable $v$ and destination $d$ if $v$ matches the source of the move. So we have the following three rules.

1. If instruction $I_k$ is an arithmetic instruction such as (addq s d), then add the edge $(d, v)$ for every $v \in L_{after}(k)$ unless $v = d$.

2. If instruction $I_k$ is of the form (callq label), then add an edge $(r, v)$ for every caller-saved register $r$ and every variable $v \in L_{after}(k)$.

3. If instruction $I_k$ is a move: (movq s d), then add the edge $(d, v)$ for every $v \in L_{after}(k)$ unless $v = d$ or $v = s$.

Working from the top to bottom of Figure 3.2, we obtain the following interference for the instruction at the specified line number.

Line 2: no interference,
Line 3: $w$ interferes with $v$,
Line 4: $x$ interferes with $w$,
Line 5: $x$ interferes with $w$,
Line 6: $y$ interferes with $w$,
Line 7: $y$ interferes with $w$ and $x$,
Line 8: $z$ interferes with $w$ and $y$,
Line 9: $z$ interferes with $y$,
Line 10: $t.1$ interferes with $z$,
Line 11: $t.1$ interferes with $z$,
Line 12: no interference,
Line 13: no interference.
Line 14: no interference.

The resulting interference graph is shown in Figure 3.3.
Exercise 10. Implement the compiler pass named `build-interference` according to the algorithm suggested above. We recommend using the Racket `graph` package to create and inspect the interference graph. The output graph of this pass should be stored in the `info` field of the program, under the key `conflicts`.

3.4 Graph Coloring via Sudoku

We now come to the main event, mapping variables to registers (or to stack locations in the event that we run out of registers). We need to make sure not to map two variables to the same register if the two variables interfere with each other. In terms of the interference graph, this means that adjacent vertices must be mapped to different registers. If we think of registers as colors, the register allocation problem becomes the widely-studied graph coloring problem \cite{Balakrishnan1996, Rosen2002}.

The reader may be more familiar with the graph coloring problem than he or she realizes; the popular game of Sudoku is an instance of the graph coloring problem. The following describes how to build a graph out of an initial Sudoku board.

- There is one vertex in the graph for each Sudoku square.
- There is an edge between two vertices if the corresponding squares are in the same row, in the same column, or if the squares are in the same $3 \times 3$ region.
- Choose nine colors to correspond to the numbers 1 to 9.
- Based on the initial assignment of numbers to squares in the Sudoku board, assign the corresponding colors to the corresponding vertices in the graph.
3.4. GRAPH COLORING VIA SUDOKU

Figure 3.4: A Sudoku game board and the corresponding colored graph.

If you can color the remaining vertices in the graph with the nine colors, then you have also solved the corresponding game of Sudoku. Figure 3.4 shows an initial Sudoku game board and the corresponding graph with colored vertices. We map the Sudoku number 1 to blue, 2 to yellow, and 3 to red. We only show edges for a sampling of the vertices (those that are colored) because showing edges for all of the vertices would make the graph unreadable.

Given that Sudoku is an instance of graph coloring, one can use Sudoku strategies to come up with an algorithm for allocating registers. For example, one of the basic techniques for Sudoku is called Pencil Marks. The idea is that you use a process of elimination to determine what numbers no longer make sense for a square, and write down those numbers in the square (writing very small). For example, if the number 1 is assigned to a square, then by process of elimination, you can write the pencil mark 1 in all the squares in the same row, column, and region. Many Sudoku computer games provide automatic support for Pencil Marks. The Pencil Marks technique corresponds to the notion of color saturation due to Brélaz [1979].

The saturation of a vertex, in Sudoku terms, is the set of colors that are no longer available. In graph terminology, we have the following definition:

\[
\text{saturation}(u) = \{ c \mid \exists v. v \in \text{neighbors}(u) \text{ and } \text{color}(v) = c \}
\]

where \( \text{neighbors}(u) \) is the set of vertices that share an edge with \( u \).

Using the Pencil Marks technique leads to a simple strategy for filling in numbers: if there is a square with only one possible number left, then
Algorithm: DSATUR
Input: a graph $G$
Output: an assignment $\text{color}[v]$ for each vertex $v \in G$

$W \leftarrow \text{vertices}(G)$

while $W \neq \emptyset$ do
    pick a vertex $u$ from $W$ with the highest saturation,
    breaking ties randomly
    find the lowest color $c$ that is not in $\{\text{color}[v] : v \in \text{adjacent}(u)\}$
    $\text{color}[u] \leftarrow c$
    $W \leftarrow W \setminus \{u\}$

Figure 3.5: The saturation-based greedy graph coloring algorithm.

write down that number! But what if there are no squares with only one possibility left? One brute-force approach is to just make a guess. If that guess ultimately leads to a solution, great. If not, backtrack to the guess and make a different guess. One good thing about Pencil Marks is that it reduces the degree of branching in the search tree. Nevertheless, backtracking can be horribly time consuming. One way to reduce the amount of backtracking is to use the most-constrained-first heuristic. That is, when making a guess, always choose a square with the fewest possibilities left (the vertex with the highest saturation). The idea is that choosing highly constrained squares earlier rather than later is better because later there may not be any possibilities.

In some sense, register allocation is easier than Sudoku because we can always cheat and add more numbers by mapping variables to the stack. We say that a variable is spilled when we decide to map it to a stack location. We would like to minimize the time needed to color the graph, and backtracking is expensive. Thus, it makes sense to keep the most-constrained-first heuristic but drop the backtracking in favor of greedy search (guess and just keep going). Figure 3.5 gives the pseudo-code for this simple greedy algorithm for register allocation based on saturation and the most-constrained-first heuristic, which is roughly equivalent to the DSATUR algorithm of Brélaz [1979] (also known as saturation degree ordering [Gebremedhin 1999] Al-Omari and Sabri 2006). Just as in Sudoku, the algorithm represents colors with integers, with the first $k$ colors corresponding to the $k$ registers in a given machine and the rest of the integers corresponding to stack locations.
With this algorithm in hand, let us return to the running example and consider how to color the interference graph in Figure 3.3. We shall not use register rax for register allocation because we use it to patch instructions, so we remove that vertex from the graph. Initially, all of the vertices are not yet colored and they are unsaturated, so we annotate each of them with a dash for their color and an empty set for the saturation.

\[
\begin{align*}
v & : -, \{ \} \\
w & : -, \{ \} \\
x & : -, \{ \} \\
y & : -, \{ \} \\
z & : -, \{ \} \\
t.1 & : -, \{ \}
\end{align*}
\]

We select a maximally saturated vertex and color it 0. In this case we have a 7-way tie, so we arbitrarily pick \( t.1 \). The then mark color 0 as no longer available for \( z \) because it interferes with \( t.1 \).

\[
\begin{align*}
v & : -, \{ \} \\
w & : -, \{ \} \\
x & : -, \{ \} \\
y & : -, \{ \} \\
z & : -, \{ 0 \} \\
t.1 & : 0, \{ \}
\end{align*}
\]

Now we repeat the process, selecting another maximally saturated vertex, which in this case is \( z \). We color \( z \) with 1.

\[
\begin{align*}
v & : -, \{ \} \\
w & : -, \{ 1 \} \\
x & : -, \{ \} \\
y & : -, \{ 1 \} \\
z & : 1, \{ 0 \} \\
t.1 & : 0, \{ 1 \}
\end{align*}
\]

The most saturated vertices are now \( w \) and \( y \). We color \( y \) with the first available color, which is 0.

\[
\begin{align*}
v & : -, \{ \} \\
w & : -, \{ 0, 1 \} \\
x & : -, \{ 0, 1 \} \\
y & : 0, \{ 1 \} \\
z & : 1, \{ 0 \} \\
t.1 & : 0, \{ 1 \}
\end{align*}
\]

Vertex \( w \) is now the most highly saturated, so we color \( w \) with 2.

\[
\begin{align*}
v & : -, \{ 2 \} \\
w & : 2, \{ 0, 1 \} \\
x & : -, \{ 0, 2 \} \\
y & : 0, \{ 1, 2 \} \\
z & : 1, \{ 0, 2 \} \\
t.1 & : 0, \{ \}
\end{align*}
\]
Now $x$ has the highest saturation, so we color it 1.

$$
v: -, \{2\} \quad w: 2, \{0, 1\} \quad x: 1, \{0, 2\} \quad y: 0, \{1, 2\} \quad z: 1, \{0, 2\} \quad t.1: 0, \{\}
$$

In the last step of the algorithm, we color $v$ with 0.

$$
v: 0, \{2\} \quad w: 2, \{0, 1\} \quad x: 1, \{0, 2\} \quad y: 0, \{1, 2\} \quad z: 1, \{0, 2\} \quad t.1: 0, \{\}
$$

With the coloring complete, we can finalize the assignment of variables to registers and stack locations. Recall that if we have $k$ registers, we map the first $k$ colors to registers and the rest to stack locations. Suppose for the moment that we have just one register to use for register allocation, rcx. Then the following is the mapping of colors to registers and stack allocations.

$$\{0 \mapsto \%rcx, 1 \mapsto -8(\%rbp), 2 \mapsto -16(\%rbp), \ldots\}$$

Putting this mapping together with the above coloring of the variables, we arrive at the assignment:

$$\{v \mapsto \%rcx, w \mapsto -16(\%rbp), x \mapsto -8(\%rbp), y \mapsto \%rcx, z \mapsto -8(\%rbp), t.1 \mapsto \%rcx\}$$

Applying this assignment to our running example, on the left, yields the program on the right.

```assembly
(block ()
  (movq (int 1) (var v))
  (movq (int 46) (var w))
  (movq (var v) (var x))
  (addq (int 7) (var x))
  (movq (var x) (var y))
  (addq (int 4) (var y))
) ⇒
(block ()
  (movq (int 1) (reg rcx))
  (movq (int 46) (deref rbp -16))
  (movq (reg rcx) (deref rbp -8))
  (addq (int 7) (deref rbp -8))
  (movq (deref rbp -8) (reg rcx))
  (addq (int 4) (reg rcx))
  (movq (deref rbp -8) (deref rbp -8))
  (addq (deref rbp -16) (deref rbp -8))
  (movq (reg rcx) (reg rcx))
  (negq (reg rax))
  (negq (reg rcx))
  (movq (deref rbp -8) (reg rax))
  (addq (reg rcx) (reg rax))
  (addq (reg rcx) (reg rax))
  (jmp conclusion))
)
The resulting program is almost an x86 program. The remaining step is to apply the patch instructions pass. In this example, the trivial move of \(-8(\%rbp)\) to itself is deleted and the addition of \(-16(\%rbp)\) to \(-8(\%rbp)\) is fixed by going through \(\text{rax}\) as follows.

\[
\begin{align*}
&\text{(movq (deref rbp -16) (reg rax))} \\
&\text{(addq (reg rax) (deref rbp -8))}
\end{align*}
\]

An overview of all of the passes involved in register allocation is shown in Figure 3.6.

**Exercise 11.** Implement the pass \texttt{allocate-registers}, which should come after the \texttt{build-interference} pass. The three new passes, \texttt{uncover-live}, \texttt{build-interference}, and \texttt{allocate-registers} replace the \texttt{assign-homes} pass of Section 2.9.

We recommend that you create a helper function named \texttt{color-graph} that takes an interference graph and a list of all the variables in the program. This function should return a mapping of variables to their colors (represented as natural numbers). By creating this helper function, you will be able to reuse it in Chapter 6 when you add support for functions.

Once you have obtained the coloring from \texttt{color-graph}, you can assign the variables to registers or stack locations and then reuse code from the \texttt{assign-homes} pass from Section 2.9 to replace the variables with their
assigned location.

Test your updated compiler by creating new example programs that exercise all of the register allocation algorithm, such as forcing variables to be spilled to the stack.

3.5 Print x86 and Conventions for Registers

Recall the print-x86 pass generates the prelude and conclusion instructions for the main function. The prelude saved the values in rbp and rsp and the conclusion returned those values to rbp and rsp. The reason for this is that our main function must adhere to the x86 calling conventions that we described in Section 3.1. In addition, the main function needs to restore (in the conclusion) any callee-saved registers that get used during register allocation. The simplest approach is to save and restore all of the callee-saved registers. The more efficient approach is to keep track of which callee-saved registers were used and only save and restore them. Either way, make sure to take this use of stack space into account when you are calculating the size of the frame. Also, don’t forget that the size of the frame needs to be a multiple of 16 bytes.

3.6 Challenge: Move Biasing*

This section describes an optional enhancement to register allocation for those students who are looking for an extra challenge or who have a deeper interest in register allocation.

We return to the running example, but we remove the supposition that we only have one register to use. So we have the following mapping of color numbers to registers.

\[
\{0 \mapsto \%rbx, \ 1 \mapsto \%rcx, \ 2 \mapsto \%rdx, \ldots\}
\]

Using the same assignment that was produced by register allocator described in the last section, we get the following program.
While this allocation is quite good, we could do better. For example, the variables \( v \) and \( x \) ended up in different registers, but if they had been placed in the same register, then the move from \( v \) to \( x \) could be removed.

We say that two variables \( p \) and \( q \) are move related if they participate together in a \texttt{movq} instruction, that is, \texttt{movq} \( p \), \( q \) or \texttt{movq} \( q \), \( p \). When the register allocator chooses a color for a variable, it should prefer a color that has already been used for a move-related variable (assuming that they do not interfere). Of course, this preference should not override the preference for registers over stack locations, but should only be used as a tie breaker when choosing between registers or when choosing between stack locations.

We recommend that you represent the move relationships in a graph, similar to how we represented interference. The following is the move graph for our running example.

Now we replay the graph coloring, pausing to see the coloring of \( x \) and \( v \). So we have the following coloring and the most saturated vertex is \( x \).

\[
\begin{align*}
v & : -, \{2\} \quad w & : 2, \{0, 1\} \quad x & : -, \{0, 2\} \\
& \quad y : 0, \{1, 2\} \quad & \quad z : 1, \{0, 2\} \quad & \quad t.1 : 0, \{\} \\
\end{align*}
\]
Last time we chose to color $x$ with 1, which so happens to be the color of $z$, and $x$ is move related to $z$. This was rather lucky, and if the program had been a little different, and say $z$ had been already assigned to 2, then $x$ would still get 1 and our luck would have run out. With move biasing, we use the fact that $x$ and $z$ are move related to influence the choice of color for $x$, in this case choosing 1 because that’s the color of $z$.

$$
v : - , \{2\} \quad w : 2 , \{0,1\} \quad x : 1 , \{0,2\}
$$

$$y : 0 , \{1,2\} \quad z : 1 , \{0,2\} \quad t.1 : 0 , \{\}
$$

Next we consider coloring the variable $v$, and we just need to avoid choosing 2 because of the interference with $w$. Last time we choose the color 0, simply because it was the lowest, but this time we know that $v$ is move related to $x$, so we choose the color 1.

$$v : 1 , \{2\} \quad w : 2 , \{0,1\} \quad x : 1 , \{0,2\}
$$

$$y : 0 , \{1,2\} \quad z : 1 , \{0,2\} \quad t.1 : 0 , \{\}
$$

We apply this register assignment to the running example, on the left, to obtain the code on right.

(block ()
(movq (int 1) (var v))
(movq (int 46) (var w))
(movq (var v) (var x))
(addq (int 7) (var x))
(movq (var x) (var y))
(addq (int 4) (var y))
(movq (var x) (var z))
(addq (var w) (var z))
(movq (var y) (var t.1))
(negq (var t.1))
(movq (var z) (reg rax))
(addq (var t.1) (reg rax))
(jmp conclusion))
⇒
(block ()
(movq (int 1) (reg rcx))
(movq (int 46) (reg rbx))
(movq (reg rcx) (reg rcx))
(addq (int 7) (reg rcx))
(movq (reg rcx) (reg rdx))
(addq (int 4) (reg rdx))
(movq (reg rcx) (reg rcx))
(addq (reg rbx) (reg rcx))
(movq (reg rdx) (reg rbx))
(negq (reg rbx))
(movq (reg rcx) (reg rax))
(addq (reg rbx) (reg rax))
(jmp conclusion))

The patch-instructions then removes the trivial moves from $v$ to $x$ and from $x$ to $z$ to obtain the following result.
Exercise 12. Change your implementation of allocate-registers to take move biasing into account. Make sure that your compiler still passes all of the previous tests. Create two new tests that include at least one opportunity for move biasing and visually inspect the output x86 programs to make sure that your move biasing is working properly.
3. REGISTER ALLOCATION
Booleans and Control Flow

The $R_0$ and $R_1$ languages only had a single kind of value, the integers. In this Chapter we add a second kind of value, the Booleans, to create the $R_2$ language. The Boolean values *true* and *false* are written $\#t$ and $\#f$ respectively in Racket. We also introduce several operations that involve Booleans (and, not, eq?, <, etc.) and the conditional if expression. With the addition of if expressions, programs can have non-trivial control flow which has an impact on several parts of the compiler. Also, because we now have two kinds of values, we need to worry about programs that apply an operation to the wrong kind of value, such as (not 1).

There are two language design options for such situations. One option is to signal an error and the other is to provide a wider interpretation of the operation. The Racket language uses a mixture of these two options, depending on the operation and the kind of value. For example, the result of (not 1) in Racket is $\#f$ because Racket treats non-zero integers like $\#t$. On the other hand, (car 1) results in a run-time error in Racket stating that car expects a pair.

The Typed Racket language makes similar design choices as Racket, except much of the error detection happens at compile time instead of run time. Like Racket, Typed Racket accepts and runs (not 1), producing $\#f$. But in the case of (car 1), Typed Racket reports a compile-time error because Typed Racket expects the type of the argument to be of the form (Listof T) or (Pairof T1 T2).

For the $R_2$ language we choose to be more like Typed Racket in that we shall perform type checking during compilation. In Chapter 8 we study the alternative choice, that is, how to compile a dynamically typed language like Racket. The $R_2$ language is a subset of Typed Racket but by no means
includes all of Typed Racket. Furthermore, for many of the operations we shall take a narrower interpretation than Typed Racket, for example, rejecting (not 1).

This chapter is organized as follows. We begin by defining the syntax and interpreter for the $R_2$ language (Section 4.1). We then introduce the idea of type checking and build a type checker for $R_2$ (Section 4.2). To compile $R_2$ we need to enlarge the intermediate language $C_0$ into $C_1$, which we do in Section 4.5. The remaining sections of this Chapter discuss how our compiler passes need to change to accommodate Booleans and conditional control flow.

4.1 The $R_2$ Language

The syntax of the $R_2$ language is defined in Figure 4.1. It includes all of $R_1$ (shown in gray), the Boolean literals #t and #f, and the conditional if expression. Also, we expand the operators to include subtraction, and, or and not, the eq? operations for comparing two integers or two Booleans, and the $<$, $\leq$, $>$, and $\geq$ operations for comparing integers.

Figure 4.2 defines the interpreter for $R_2$, omitting the parts that are the same as the interpreter for $R_1$ (Figure 2.2). The literals #t and #f simply evaluate to themselves. The conditional expression (if cnd thn els) evaluates the Boolean expression cnd and then either evaluates thn or els depending on whether cnd produced #t or #f. The logical operations not and and behave as you might expect, but note that the and operation is short-circuiting. That is, given the expression (and e1 e2), the expression e2 is not evaluated if e1 evaluates to #f.

With the addition of the comparison operations, there are quite a few primitive operations and the interpreter code for them is somewhat repetitive. In Figure 4.2 we factor out the different parts into the interp-op

\[
\begin{align*}
\text{cmp} &::= \text{eq?} | < | \leq | > | \\
\text{exp} &::= \text{int} | (\text{read}) | (\neg \text{exp}) | (+ \text{exp} \text{exp}) | (- \text{exp} \text{exp}) \\
&\quad | \text{var} | (\text{let} ([\text{var} \text{exp}]) \text{exp}) \\
&\quad | \#t | \#f | (\text{and} \text{exp} \text{exp}) | (\text{or} \text{exp} \text{exp}) | (\text{not} \text{exp}) \\
&\quad | (\text{cmp} \text{exp} \text{exp}) | (\text{if} \text{exp} \text{exp} \text{exp}) \\
\text{R}_2 &::= (\text{program info exp})
\end{align*}
\]

Figure 4.1: The syntax of $R_2$, extending $R_1$ (Figure 2.1) with Booleans and conditionals.
function and the similar parts into the one match clause shown in Figure 4.2. We do not use interp-op for the and operation because of the short-circuiting behavior in the order of evaluation of its arguments.

4.2 Type Checking $R_2$ Programs

It is helpful to think about type checking in two complementary ways. A type checker predicts the type of value that will be produced by each expression in the program. For $R_2$, we have just two types, Integer and Boolean. So a type checker should predict that

$$(+ 10 (- (+ 12 20)))$$

produces an Integer while

$$(\text{and} (\text{not} \ #f) \ #t)$$

produces a Boolean.

As mentioned at the beginning of this chapter, a type checker also rejects programs that apply operators to the wrong type of value. Our type checker for $R_2$ will signal an error for the following expression because, as we have seen above, the expression $(+ 10 \ldots)$ has type Integer, and we require the argument of a not to have type Boolean.

$$(\text{not} (+ 10 (- (+ 12 20))))$$

The type checker for $R_2$ is best implemented as a structurally recursive function over the AST. Figure 4.3 shows many of the clauses for the type-check-exp function. Given an input expression $e$, the type checker either returns the type (Integer or Boolean) or it signals an error. Of course, the type of an integer literal is Integer and the type of a Boolean literal is Boolean. To handle variables, the type checker, like the interpreter, uses an association list. However, in this case the association list maps variables to types instead of values. Consider the clause for let. We type check the initializing expression to obtain its type $T$ and then associate type $T$ with the variable $x$. When the type checker encounters the use of a variable, it can find its type in the association list.

Exercise 13. Complete the implementation of type-check-R2 and test it on 10 new example programs in $R_2$ that you choose based on how thoroughly they test the type checking algorithm. Half of the example programs should have a type error, to make sure that your type checker properly rejects them. The other half of the example programs should not have type errors. Your testing should check that the result of the type checker agrees with
(define (interp-op op)
  (match op
    ...
    ['not (lambda (v) (match v [#t #f] [#f #t]))]
    ['eq? (lambda (v1 v2)
      (cond [(or (and (fixnum? v1) (fixnum? v2))
               (and (boolean? v1) (boolean? v2)))
             (eq? v1 v2)])]
    ['< (lambda (v1 v2)
      (cond [(and (fixnum? v1) (fixnum? v2)) (< v1 v2)])]
    ['<= (lambda (v1 v2)
      (cond [(and (fixnum? v1) (fixnum? v2)) (<= v1 v2)])]
    ['> (lambda (v1 v2)
      (cond [(and (fixnum? v1) (fixnum? v2)) (> v1 v2)])]
    ['>= (lambda (v1 v2)
      (cond [(and (fixnum? v1) (fixnum? v2)) (>= v1 v2)])]
    [else (error 'interp-op "unknown binary operator")]))

(define (interp-exp env)
  (lambda (e)
    (define recur (interp-exp env))
    (match e
      ...
      ['? boolean? e]
      ['if ,cnd ,thn ,els)
        (define b (recur cnd))
        (match b
          [#t (recur thn)]
          [#f (recur els)])]
      ['and ,e1 ,e2)
        (define v1 (recur e1))
        (match v1
          [#t (match (recur e2) [#t [#t] [#f #f]])
            [#f #f]]])
      [else (error 'interp-op "unknown binary operator")])
    ))

(define (interp-R2 env)
  (lambda (p)
    (match p
      ['program ,info ,e)
        ((interp-exp '()) e)]))

Figure 4.2: Interpreter for the $R_2$ language.
(define (type-check-exp env)
  (lambda (e)
    (define recur (type-check-exp env))
    (match e
      [(? fixnum?) 'Integer]
      [(? boolean?) 'Boolean]
      [(? symbol? x) (dict-ref env x)]
      ['(read) 'Integer]
      ['(let ([,x ,e]) ,body)
        (define T (recur e))
        (define new-env (cons (cons x T) env))
        (type-check-exp new-env body)]
      ...
      ['(not ,e)
        (match (recur e)
          ['Boolean 'Boolean]
          [else (error 'type-check-exp "'not' expects a Boolean" e)]]
      ...
    )))

(define (type-check-R2 env)
  (lambda (e)
    (match e
      ['(program ,info ,body)
        (define ty ((type-check-exp '()) body))
        ('(program ,info ,body)
          ))))

Figure 4.3: Skeleton of a type checker for the $R_2$ language.
the value returned by the interpreter, that is, if the type checker returns \texttt{Integer}, then the interpreter should return an integer. Likewise, if the type checker returns \texttt{Boolean}, then the interpreter should return \texttt{#t} or \texttt{#f}. Note that if your type checker does not signal an error for a program, then interpreting that program should not encounter an error. If it does, there is something wrong with your type checker.

4.3 Shrink the $R_2$ Language

The $R_2$ language includes several operators that are easily expressible in terms of other operators. For example, subtraction is expressible in terms of addition and negation.

\[( - e_1 e_2 ) \Rightarrow ( + e_1 (- e_2) )\]

Several of the comparison operations are expressible in terms of less-than and logical negation.

\[( <= e_1 e_2 ) \Rightarrow (\text{not} ( < e_2 e_1 ) )\]

By performing these translations near the front-end of the compiler, the later passes of the compiler will not need to deal with these constructs, making those passes shorter. On the other hand, sometimes these translations make it more difficult to generate the most efficient code with respect to the number of instructions. However, these differences typically do not affect the number of accesses to memory, which is the primary factor that determines execution time on modern computer architectures.

\textbf{Exercise 14.} Implement the pass \texttt{shrink} that removes subtraction, \texttt{and}, \texttt{or}, \texttt{<=}, \texttt{>,}, and \texttt{>=} from the language by translating them to other constructs in $R_2$. Create tests to make sure that the behavior of all of these constructs stays the same after translation.

4.4 XOR, Comparisons, and Control Flow in x86

To implement the new logical operations, the comparison operations, and the \texttt{if} expression, we need to delve further into the x86 language. Figure 4.4 defines the abstract syntax for a larger subset of x86 that includes instructions for logical operations, comparisons, and jumps.

One small challenge is that x86 does not provide an instruction that directly implements logical negation (\texttt{not} in $R_2$ and $C_1$). However, the
xorq instruction can be used to encode not. The xorq instruction takes two arguments, performs a pairwise exclusive-or operation on each bit of its arguments, and writes the results into its second argument. Recall the truth table for exclusive-or:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

For example, 0011 XOR 0101 = 0110. Notice that in row of the table for the bit 1, the result is the opposite of the second bit. Thus, the not operation can be implemented by xorq with 1 as the first argument: 0001 XOR 0000 = 0001 and 0001 XOR 0001 = 0000.

Next we consider the x86 instructions that are relevant for compiling the comparison operations. The cmpq instruction compares its two arguments to determine whether one argument is less than, equal, or greater than the other argument. The cmpq instruction is unusual regarding the order of its arguments and where the result is placed. The argument order is backwards: if you want to test whether \( x < y \), then write cmpq \( y, x \). The result of cmpq is placed in the special EFLAGS register. This register cannot be accessed directly but it can be queried by a number of instructions, including the set instruction. The set instruction puts a 1 or 0 into its destination depending on whether the comparison came out according to the condition code \( cc \) (e for equal, 1 for less, le for less-or-equal, g for greater, ge for greater-or-equal). The set instruction has an annoying quirk in that its destination argument must be single byte register, such as al, which is part of the rax register. Thankfully, the movzbq instruction can then be used to move from a single byte register to a normal 64-bit register.
4. Booleans and Control Flow

<table>
<thead>
<tr>
<th>arg</th>
<th>::= int</th>
<th>var</th>
<th>#t</th>
<th>#f</th>
</tr>
</thead>
<tbody>
<tr>
<td>cmp</td>
<td>::= eq?</td>
<td>&lt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exp</td>
<td>::= arg</td>
<td>(read)</td>
<td>(- arg)</td>
<td>(+ arg arg)</td>
</tr>
<tr>
<td>stmt</td>
<td>::= (assign var exp)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tail</td>
<td>::= (return exp)</td>
<td>(seq stmt tail)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(goto label)</td>
<td>(if (cmp arg arg) (goto label) (goto label))</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>::= (program info ((label . tail) ^))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.5: The $C_1$ language, extending $C_0$ with Booleans and conditionals.

For compiling the if expression, the x86 instructions for jumping are relevant. The jmp instruction updates the program counter to point to the instruction after the indicated label. The jmp-if instruction updates the program counter to point to the instruction after the indicated label depending on whether the result in the EFLAGS register matches the condition code $cc$, otherwise the jmp-if instruction falls through to the next instruction. Because the jmp-if instruction relies on the EFLAGS register, it is quite common for the jmp-if to be immediately preceded by a cmpq instruction, to set the EFLAGS register. Our abstract syntax for jmp-if differs from the concrete syntax for x86 to separate the instruction name from the condition code. For example, (jmp-if le foo) corresponds to jle foo.

4.5 The $C_1$ Intermediate Language

As with $R_1$, we shall compile $R_2$ to a C-like intermediate language, but we need to grow that intermediate language to handle the new features in $R_2$: Booleans and conditional expressions. Figure 4.5 shows the new features of $C_1$; we add logic and comparison operators to the exp non-terminal, the literals #t and #f to the arg non-terminal. Regarding control flow, $C_1$ differs considerably from $R_2$. Instead of if expressions, $C_1$ has goto's and conditional goto's in the grammar for tail. This means that a sequence of statements may now end with a goto or a conditional goto, which jumps to one of two labeled pieces of code depending on the outcome of the comparison. In Section 4.6, we discuss how to translate from $R_2$ to $C_1$, bridging this gap between if expressions and goto's.
4.6 Explicate Control

Recall that the purpose of explicate-control is to make the order of evaluation explicit in the syntax of the program. With the addition of if in $R_2$, things get more interesting.

As a motivating example, consider the following program that has an if expression nested in the predicate of another if.

```
(program ()
  (if (if (eq? (read) 1)
       (eq? (read) 0)
       (eq? (read) 2))
     (+ 10 32)
     (+ 700 77)))
```

The naive way to compile if and eq? would be to handle each of them in isolation, regardless of their context. Each eq? would be translated into a cmpq instruction followed by a couple instructions to move the result from the EFLAGS register into a general purpose register or stack location. Each if would be translated into the combination of a cmpq and jmp-if. However, if we take context into account we can do better and reduce the use of cmpq and EFLAG-accessing instructions.

One idea is to try and reorganize the code at the level of $R_2$, pushing the outer if inside the inner one. This would yield the following code.

```
(if (eq? (read) 1)
  (if (eq? (read) 0)
    (+ 10 32)
    (+ 700 77))
  (if (eq? (read) 2))
  (+ 10 32)
  (+ 700 77))
```

Unfortunately, this approach duplicates the two branches, and a compiler must never duplicate code!

We need a way to perform the above transformation, but without duplicating code. The solution is straightforward if we think at the level of x86 assembly: we can label the code for each of the branches and insert goto's in all the places that need to execute the branches. Put another way, we need to move away from abstract syntax trees and instead use graphs. In particular, we shall use a standard program representation called a control flow graph (CFG), due to Frances Elizabeth Allen [1970]. Each vertex is a labeled sequence of code, called a basic block, and each edge represents a jump to another block. The program construct of $C_0$ and $C_1$ represents
a control flow graph as an association list mapping labels to basic blocks. Each block is represented by the *tail* non-terminal.

Figure 4.6 shows the output of the *remove-complex-opera* pass and then the *explicate-control* pass on the example program. We shall walk through the output program and then discuss the algorithm. Following the order of evaluation in the output of *remove-complex-opera*, we first have the *(read)* and comparison to 1 from the predicate of the inner *if*. In the output of *explicate-control*, in the *start* block, this becomes a *(read)* followed by a conditional goto to either *block61* or *block62*. Each of these contains the translations of the code *(eq? (read) 0)* and *(eq? (read) 1)*, respectively. Regarding *block61*, we start with the *(read)* and comparison to 0 and then have a conditional goto, either to *block59* or *block60*, which indirectly take us to *block55* and *block56*, the two branches of the outer *if*, i.e., *(+ 10 32)* and *(+ 700 77)*. The story for *block62* is similar.

The nice thing about the output of *explicate-control* is that there

Figure 4.6: Example translation from $R_2$ to $C_1$ via the *explicate-control*. 
are no unnecessary uses of \texttt{eq?} and every use of \texttt{eq?} is part of a conditional jump. The down-side of this output is that it includes trivial blocks, such as block57 through block60, that only jump to another block. We discuss a solution to this problem in Section 4.11.

Recall that in Section 2.6 we implement the \texttt{explicate-control} pass for \textit{R}$_1$ using two mutually recursive functions, \texttt{explicate-control-tail} and \texttt{explicate-control-assign}. The former function translated expressions in tail position whereas the later function translated expressions on the right-hand-side of a \texttt{let}. With the addition of \texttt{if} expression in \textit{R}$_2$ we have a new kind of context to deal with: the predicate position of the \texttt{if}. So we shall need another function, \texttt{explicate-control-pred}, that takes an \textit{R}$_2$ expression and two pieces of \textit{C}$_1$ code (two tail’s) for the then-branch and else-branch. The output of \texttt{explicate-control-pred} is a \textit{C}$_1$ \textit{tail}. However, these three functions also need to construct the control-flow graph, which we recommend they do via updates to a global variable. Next we consider the specific additions to the tail and assign functions, and some of cases for the \texttt{pred} function.

The \texttt{explicate-control-tail} function needs an additional case for \texttt{if}. The branches of the \texttt{if} inherit the current context, so they are in tail position. Let $B_1$ be the result of \texttt{explicate-control-tail} on the \texttt{thn} branch and $B_2$ be the result of apply \texttt{explicate-control-tail} to the \texttt{els} branch. Then the \texttt{if} translates to the block $B_3$ which is the result of applying \texttt{explicate-control-pred} to the predicate \texttt{cnd} and the blocks $B_1$ and $B_2$.

\[
\texttt{(if cnd thn els) } \Rightarrow \quad B_3
\]

Next we consider the case for \texttt{if} in the \texttt{explicate-control-assign} function. So the context of the \texttt{if} is an assignment to some variable \texttt{x} and then the control continues to some block $B_1$. The code that we generate for both the \texttt{thn} and \texttt{els} branches shall both need to continue to $B_1$, so we add $B_1$ to the control flow graph with a fresh label $\ell_1$. Again, the branches of the \texttt{if} inherit the current context, so that are in assignment positions. Let $B_2$ be the result of applying \texttt{explicate-control-assign} to the \texttt{thn} branch, variable \texttt{x}, and the block (\texttt{goto} $\ell_1$). Let $B_3$ be the result of applying \texttt{explicate-control-assign} to the \texttt{else} branch, variable \texttt{x}, and the block (\texttt{goto} $\ell_1$). The \texttt{if} translates to the block $B_4$ which is the result of applying \texttt{explicate-control-pred} to the predicate \texttt{cnd} and the blocks $B_2$ and $B_3$.

\[
\texttt{(if cnd thn els) } \Rightarrow \quad B_4
\]

The function \texttt{explicate-control-pred} will need a case for every expression that can have type \texttt{Boolean}. We detail a few cases here and leave
the rest for the reader. The input to this function is an expression and two blocks, \(B_1\) and \(B_2\), for the branches of the enclosing \textit{if}. One of the base cases of this function is when the expression is a less-than comparison. We translate it to a conditional \texttt{goto}. We need labels for the two branches \(B_1\) and \(B_2\), so we add them to the control flow graph and obtain some labels \(\ell_1\) and \(\ell_2\). The translation of the less-than comparison is as follows.

\[
(< e_1 e_2) \Rightarrow (\text{if} (< e_1 e_2) (\text{goto} \ \ell_1) (\text{goto} \ \ell_2))
\]

The case for \texttt{if} in \texttt{explicate-control-pred} is particularly illuminating, as it deals with the challenges that we discussed above regarding the example of the nested \texttt{if} expressions. Again, we add the two input branches \(B_1\) and \(B_2\) to the control flow graph and obtain the labels \(\ell_1\) and \(\ell_2\). The branches \texttt{thn} and \texttt{els} of the current \texttt{if} inherit their context from the current one, i.e., predicate context. So we apply \texttt{explicate-control-pred} to \texttt{thn} with the two blocks \((\text{goto} \ \ell_1)\) and \((\text{goto} \ \ell_2)\), to obtain \(B_3\). Similarly for the \texttt{els} branch, to obtain \(B_4\). Finally, we apply \texttt{explicate-control-pred} to the predicate \texttt{cnd} and the blocks \(B_3\) and \(B_4\) to obtain the result \(B_5\).

\[
(\texttt{if} \ \texttt{cnd} \ \texttt{thn} \ \texttt{els}) \Rightarrow B_5
\]

**Exercise 15.** Implement the pass \texttt{explicate-code} by adding the cases for \texttt{if} to the functions for tail and assignment contexts, and implement the function for predicate contexts. Create test cases that exercise all of the new cases in the code for this pass.

### 4.7 Select Instructions

Recall that the \texttt{select-instructions} pass lowers from our \texttt{C}-like intermediate representation to the pseudo-x86 language, which is suitable for conducting register allocation. The pass is implemented using three auxiliary functions, one for each of the non-terminals \texttt{arg}, \texttt{stmt}, and \texttt{tail}.

For \texttt{arg}, we have new cases for the Booleans. We take the usual approach of encoding them as integers, with true as 1 and false as 0.

\[
\#\texttt{t} \Rightarrow 1 \quad \#\texttt{f} \Rightarrow 0
\]

For \texttt{stmt}, we discuss a couple cases. The \texttt{not} operation can be implemented in terms of \texttt{xorq} as we discussed at the beginning of this section. Given an assignment \((\texttt{assign} \ \texttt{lhs} \ (\texttt{not} \ \texttt{arg}))\), if the left-hand side \texttt{lhs} is the same as \texttt{arg}, then just the \texttt{xorq} suffices:

\[
(\text{assign} \ x (\text{not} \ x)) \Rightarrow ((\text{xorq} \ (\text{int} \ 1) \ x'))
\]
Otherwise, a `movq` is needed to adapt to the update-in-place semantics of x86. Let `arg'` be the result of recursively processing `arg`. Then we have

\[(\text{assign } lhs (\text{not } arg)) \Rightarrow ((\text{movq } arg' \ lhs')) (\text{xorq } (\text{int } 1) \ lhs'))\]

Next consider the cases for `eq?` and less-than comparison. Translating these operations to x86 is slightly involved due to the unusual nature of the `cmpq` instruction discussed above. We recommend translating an assignment from `eq?` into the following sequence of three instructions.

\[(\text{assign } lhs (\text{eq? } arg_1 \ arg_2)) \Rightarrow (\text{cmpq } arg'_2 \ arg'_1) (\text{set } e (\text{byte-reg al})) (\text{movzbq } (\text{byte-reg al}) \ lhs')\]

Regarding the tail non-terminal, we have two new cases, for `goto` and conditional `goto`. Both are straightforward to handle. A `goto` becomes a jump instruction.

\[(\text{goto } \ell) \Rightarrow (\text{jmp } \ell)\]

A conditional `goto` becomes a compare instruction followed by a conditional jump (for “then”) and the fall-through is to a regular jump (for “else”).

\[
\begin{align*}
(\text{if } (\text{eq? } arg_1 \ arg_2)) & \Rightarrow ((\text{cmpq } arg'_2 \ arg'_1)) \\
(\text{goto } \ell_1) & \Rightarrow (\text{jmp-if } e \ \ell_1) \\
(\text{goto } \ell_2)) & \Rightarrow (\text{jmp } \ell_2))
\end{align*}
\]

**Exercise 16.** Expand your `select-instructions` pass to handle the new features of the $R_2$ language. Test the pass on all the examples you have created and make sure that you have some test programs that use the `eq?` and `<` operators, creating some if necessary. Test the output using the `interp-x86` interpreter (Appendix 12.1).

**4.8 Register Allocation**

The changes required for $R_2$ affect the liveness analysis, building the interference graph, and assigning homes, but the graph coloring algorithm itself does not need to change.

**4.8.1 Liveness Analysis**

Recall that for $R_1$ we implemented liveness analysis for a single basic block (Section 3.2). With the addition of `if` expressions to $R_2$, `explicate-control`
now produces many basic blocks arranged in a control-flow graph. The first question we need to consider is in what order should we process the basic blocks? Recall that to perform liveness analysis, we need to know the live-after set. If a basic block has no successor blocks, then it has an empty live-after set and we can immediately apply liveness analysis to it. If a basic block has some successors, then we need to complete liveness analysis on those blocks first. Furthermore, we know that the control flow graph does not contain any cycles (it is a DAG, that is, a directed acyclic graph). What all this amounts to is that we need to process the basic blocks in reverse topological order. We recommend using the \texttt{tsort} and \texttt{transpose} functions of the Racket \texttt{graph} package to obtain this ordering.

The next question is how to compute the live-after set of a block given the live-before sets of all its successor blocks. During compilation we do not know which way the branch will go, so we do not know which of the successor’s live-before set to use. The solution comes from the observation that there is no harm in identifying more variables as live than absolutely necessary. Thus, we can take the union of the live-before sets from all the successors to be the live-after set for the block. Once we have computed the live-after set, we can proceed to perform liveness analysis on the block just as we did in Section 3.2.

The helper functions for computing the variables in an instruction’s argument and for computing the variables read-from (\textit{R}) or written-to (\textit{W}) by an instruction need to be updated to handle the new kinds of arguments and instructions in x86$^1$.

\subsection*{4.8.2 Build Interference}

Many of the new instructions in x86$^1$ can be handled in the same way as the instructions in x86$^0$. Thus, if your code was already quite general, it will not need to be changed to handle the new instructions. If not, I recommend that you change your code to be more general. The \texttt{movzbq} instruction should be handled like the \texttt{movq} instruction.

**Exercise 17.** Update the \texttt{register-allocation} pass so that it works for \textit{R}$_2$ and test your compiler using your previously created programs on the \texttt{interp-x86} interpreter (Appendix 12.1).

\footnotetext[1]{If we were to add loops to the language, then the CFG could contain cycles and we would instead need to use the classic worklist algorithm for computing the fixed point of the liveness analysis [Aho et al. 1986].}
4.9 Patch Instructions

The second argument of the `cmpq` instruction must not be an immediate value (such as a literal integer). So if you are comparing two immediates, we recommend inserting a `movq` instruction to put the second argument in `rax`. The second argument of the `movzbq` must be a register. There are no special restrictions on the x86 instructions `jmp-if`, `jmp`, and `label`.

**Exercise 18.** Update `patch-instructions` to handle the new x86 instructions. Test your compiler using your previously created programs on the `interp-x86` interpreter (Appendix 12.1).

4.10 An Example Translation

Figure 4.7 shows a simple example program in $R_2$ translated to x86, showing the results of `explicate-control`, `select-instructions`, and the final x86 assembly code.

Figure 4.8 lists all the passes needed for the compilation of $R_2$.

4.11 Challenge: Optimize Jumps*

UNDER CONSTRUCTION
4. BOOLEANS AND CONTROL FLOW

(program ()
  (if (eq? (read) 1) 42 0))

(program ()
  ((block32 . (return 0))
   (block31 . (return 42))
   (start .
    (seq (assign tmp30 (read))
         (if (eq? tmp30 1)
             (goto block31)
             (goto block32))))))

(program ((locals . (tmp30)))
  ((block32 .
     (block ()
      (movq (int 0) (reg rax))
      (jmp conclusion)))
   (block31 .
    (block ()
     (movq (int 42) (reg rax))
     (jmp conclusion)))
   (start .
    (block ()
     (callq read_int)
     (movq (reg rax) (var tmp30))
     (cmpq (int 1) (var tmp30))
     (jmp-if e block31)
     (jmp block32))))))

.mainloop:
  pushq %rbp
  movq %rsp, %rbp
  pushq %r12
  pushq %rbx
  pushq %r13
  pushq $0, %rsp
  jmp _start

_conclusion:
  addq $0, %rsp
  popq %r14
  popq %r13
  popq %rbx
  popq %r12
  popq %rbp
  retq

_block31:
  movq $42, %rax
  jmp _conclusion

_block32:
  movq $0, %rax
  jmp _conclusion

_start:
  callq _read_int
  movq %rax, %rcx
  cmpq $1, %rcx
  je _block31
  jmp _block32

.globl _main

_main:
  pushq %rbp
  movq %rsp, %rbp
  pushq %r12
  pushq %rbx
  pushq %r13
  pushq $0, %rsp
  jmp _start

_conclusion:
  addq $0, %rsp
  popq %r14
  popq %r13
  popq %rbx
  popq %r12
  popq %rbp
  retq

Figure 4.7: Example compilation of an if expression to x86.
4.11. CHALLENGE: OPTIMIZE JUMPS

Figure 4.8: Diagram of the passes for $R^*_2$, a language with conditionals.
4. BOOLEANS AND CONTROL FLOW
In this chapter we study the implementation of mutable tuples (called “vectors” in Racket). This language feature is the first to use the computer’s heap because the lifetime of a Racket tuple is indefinite, that is, a tuple lives forever from the programmer’s viewpoint. Of course, from an implementor’s viewpoint, it is important to reclaim the space associated with a tuple when it is no longer needed, which is why we also study garbage collection techniques in this chapter.

Section 5.1 introduces the $R_3$ language including its interpreter and type checker. The $R_3$ language extends the $R_2$ language of Chapter 4 with vectors and Racket’s “void” value. The reason for including the later is that the $vector-set!$ operation returns a value of type $Void$.\footnote{This may sound contradictory, but Racket’s $Void$ type corresponds to what is more commonly called the $Unit$ type. This type is inhabited by a single value that is usually written $unit$ or $()$ [Pierce 2002].}

Section 5.2 describes a garbage collection algorithm based on copying live objects back and forth between two halves of the heap. The garbage collector requires coordination with the compiler so that it can see all of the root pointers, that is, pointers in registers or on the procedure call stack. Sections 5.3 through 5.8 discuss all the necessary changes and additions to the compiler passes, including a new compiler pass named $expose-allocation$. 

(let ([t (vector 40 #t (vector 2))])
  (if (vector-ref t 1)
      (+ (vector-ref t 0)
          (vector-ref (vector-ref t 2) 0))
      44))

Figure 5.1: Example program that creates tuples and reads from them.

(type ::= Integer | Boolean | (Vector type+) | Void

(cmp ::= eq? | < | <= | > | >=

(exp ::= int | (read) | (- exp) | (+ exp exp) | (- exp exp)

          | var | (let ([var exp]) exp)

          | #t | #f | (and exp exp) | (or exp exp) | (not exp)

          | (cmp exp exp) | (if exp exp exp)

          | (vector exp+) | (vector-ref exp int)

          | (vector-set! exp int exp)

          | (void)

R₃ ::= (program exp)

Figure 5.2: The syntax of R₃, extending R₂ (Figure 4.1) with tuples.

5.1 The R₃ Language

Figure 5.2 defines the syntax for R₃, which includes three new forms for creating a tuple, reading an element of a tuple, and writing to an element of a tuple. The program in Figure 5.1 shows the usage of tuples in Racket. We create a 3-tuple \( t \) and a 1-tuple. The 1-tuple is stored at index 2 of the 3-tuple, demonstrating that tuples are first-class values. The element at index 1 of \( t \) is \#t, so the “then” branch is taken. The element at index 0 of \( t \) is 40, to which we add the 2, the element at index 0 of the 1-tuple.

Tuples are our first encounter with heap-allocated data, which raises several interesting issues. First, variable binding performs a shallow-copy when dealing with tuples, which means that different variables can refer to the same tuple, i.e., different variables can be aliases for the same thing. Consider the following example in which both \( t₁ \) and \( t₂ \) refer to the same tuple. Thus, the mutation through \( t₂ \) is visible when referencing the tuple from \( t₁ \), so the result of this program is 42.

(let ([t1 (vector 3 7)])
  (let ([t2 t1]))

(= 42)
5.2. GARBAGE COLLECTION

(let ([t (vector 3 7)]
    (vector-ref t 0))

The next issue concerns the lifetime of tuples. Of course, they are created by the vector form, but when does their lifetime end? Notice that the grammar in Figure 5.2 does not include an operation for deleting tuples. Furthermore, the lifetime of a tuple is not tied to any notion of static scoping. For example, the following program returns 3 even though the variable t goes out of scope prior to accessing the vector.

(vector-ref (let ([t (vector 3 7)])
            t) 0)

From the perspective of programmer-observable behavior, tuples live forever. Of course, if they really lived forever, then many programs would run out of memory. A Racket implementation must therefore perform automatic garbage collection.

Figure 5.3 shows the definitional interpreter for the R3 language and Figure 5.4 shows the type checker. The additions to the interpreter are straightforward but the updates to the type checker deserve some explanation. As we shall see in Section 5.2, we need to know which variables are pointers into the heap, that is, which variables are vectors. Also, when allocating a vector, we shall need to know which elements of the vector are pointers. We can obtain this information during type checking and when we uncover local variables. The type checker in Figure 5.4 not only computes the type of an expression, it also wraps every sub-expression e with the form (has-type e T), where T is e’s type. Subsequently, in the uncover-locals pass (Section 5.5) this type information is propagated to all variables (including the temporaries generated by remove-complex-opera*).

5.2 Garbage Collection

Here we study a relatively simple algorithm for garbage collection that is the basis of state-of-the-art garbage collectors [Lieberman and Hewitt, 1983, Ungar, 1984, Jones and Lins, 1996, Detlefs et al., 2004, Dybvig, 2006, Tene et al., 2011]. In particular, we describe a two-space copying collector [Wilson, 1992] that uses Cheney’s algorithm to perform the copy [Cheney, 1970].

\footnote{The R3 language does not have looping or recursive function, so it is nigh impossible to write a program in R3 that will run out of memory. However, we add recursive functions in the next Chapter!}
(define primitives (set ... 'vector 'vector-ref 'vector-set!))

(define (interp-op op)
  (match op
    ...
    ['vector vector]
    ['vector-ref vector-ref]
    ['vector-set! vector-set!]
    [else (error 'interp-op "unknown operator")]))

(define (interp-R3 env)
  (lambda (e)
    (match e
      ...
    [else (error 'interp-R3 "unrecognized expression")]
  )))

Figure 5.3: Interpreter for the $R_3$ language.

Figure 5.5 gives a coarse-grained depiction of what happens in a two-space collector, showing two time steps, prior to garbage collection on the top and after garbage collection on the bottom. In a two-space collector, the heap is divided into two parts, the FromSpace and the ToSpace. Initially, all allocations go to the FromSpace until there is not enough room for the next allocation request. At that point, the garbage collector goes to work to make more room.

The garbage collector must be careful not to reclaim tuples that will be used by the program in the future. Of course, it is impossible in general to predict what a program will do, but we can overapproximate the will-be-used tuples by preserving all tuples that could be accessed by any program given the current computer state. A program could access any tuple whose address is in a register or on the procedure call stack. These addresses are called the root set. In addition, a program could access any tuple that is transitively reachable from the root set. Thus, it is safe for the garbage collector to reclaim the tuples that are not reachable in this way.

So the goal of the garbage collector is twofold:

1. preserve all tuple that are reachable from the root set via a path of pointers, that is, the live tuples, and

2. reclaim the memory of everything else, that is, the garbage.
(define (type-check-exp env)
  (lambda (e)
    (define recur (type-check-exp env))
    (match e ...
      ['(void) (values '(has-type (void) Void)' Void)]
      ['(vector ,es ...)
        (define-values (e* t*) (for/lists (e* t*) ([e es])
            (recur e)))
        (let ([t '(Vector ,@t*)])
          (debug "vector/type-check-exp\_finished\_vector" t)
          (values '(has-type (vector ,@e*) ,t) t))]
      ['(vector-ref ,e ,i)
        (define-values (e^ t) (recur e))
        (match t ...
          ['(Vector ,ts ...)
            (unless (and (exact-nonnegative-integer? i) (< i (length ts)))
              (error 'type-check-exp "invalid/uni2423index/uni2423~a" i))
            (let ([t (list-ref ts i)])
              (values '(has-type (vector-ref ,e^ (has-type ,i Integer)) ,t) t))]
          [else (error "expected/uni2423a/uni2423vector/uni2423in/uni2423vector-ref,/uni2423not" t)]])
      ['(eq? ,arg1 ,arg2)
        (define-values (e1 t1) (recur arg1))
        (define-values (e2 t2) (recur arg2))
        (match* (t1 t2)
          ['(Vector ,ts1 ...) '(Vector ,ts2 ...)
            (values '(has-type (eq? ,e1 ,e2 Boolean) 'Boolean)]
          [(other wise) ((super type-check-exp env) e)])]
      ...))
))}

Figure 5.4: Type checker for the $R_3$ language.
A copying collector accomplishes this by copying all of the live objects from the FromSpace into the ToSpace and then performs a slight of hand, treating the ToSpace as the new FromSpace and the old FromSpace as the new ToSpace. In the example of Figure 5.5, there are three pointers in the root set, one in a register and two on the stack. All of the live objects have been copied to the ToSpace (the right-hand side of Figure 5.5) in a way that preserves the pointer relationships. For example, the pointer in the register still points to a 2-tuple whose first element is a 3-tuple and second element is a 2-tuple. There are four tuples that are not reachable from the root set and therefore do not get copied into the ToSpace. (The situation in Figure 5.5 with a cycle, cannot be created by a well-typed program in \( R_3 \). However, creating cycles will be possible once we get to \( R_6 \). We design the garbage collector to deal with cycles to begin with, so we will not need to revisit this issue.)

There are many alternatives to copying collectors (and their older siblings, the generational collectors) when it comes to garbage collection, such as mark-and-sweep and reference counting. The strengths of copying collectors are that allocation is fast (just a test and pointer increment), there is no fragmentation, cyclic garbage is collected, and the time complexity of collection only depends on the amount of live data, and not on the amount of garbage \[ \text{Wilson, 1992} \]. The main disadvantage of two-space copying collectors is that they use a lot of space, though that problem is ameliorated in generational collectors. Racket and Scheme programs tend to allocate many small objects and generate a lot of garbage, so copying and generational collectors are a good fit. Of course, garbage collection is an active research topic, especially concurrent garbage collection \[ \text{Tene et al., 2011} \]. Researchers are continuously developing new techniques and revisiting old trade-offs \[ \text{Blackburn et al., 2004; Jones et al., 2011; Shahriyar et al., 2013; Cutler and Morris, 2015; Shidal et al., 2015} \].

### 5.2.1 Graph Copying via Cheney’s Algorithm

Let us take a closer look at how the copy works. The allocated objects and pointers can be viewed as a graph and we need to copy the part of the graph that is reachable from the root set. To make sure we copy all of the reachable vertices in the graph, we need an exhaustive graph traversal algorithm, such as depth-first search or breadth-first search \[ \text{Moore, 1959; Cormen et al., 2001} \]. Recall that such algorithms take into account the possibility of cycles by marking which vertices have already been visited, so as to ensure termination of the algorithm. These search algorithms also use
Figure 5.5: A copying collector in action.
a data structure such as a stack or queue as a to-do list to keep track of the vertices that need to be visited. We shall use breadth-first search and a trick due to [Cheney 1970] for simultaneously representing the queue and copying tuples into the ToSpace.

Figure 5.6 shows several snapshots of the ToSpace as the copy progresses. The queue is represented by a chunk of contiguous memory at the beginning of the ToSpace, using two pointers to track the front and the back of the queue. The algorithm starts by copying all tuples that are immediately reachable from the root set into the ToSpace to form the initial queue. When we copy a tuple, we mark the old tuple to indicate that it has been visited. (We discuss the marking in Section 5.2.2.) Note that any pointers inside the copied tuples in the queue still point back to the FromSpace. Once the initial queue has been created, the algorithm enters a loop in which it repeatedly processes the tuple at the front of the queue and pops it off the queue. To process a tuple, the algorithm copies all the tuple that are directly reachable from it to the ToSpace, placing them at the back of the queue. The algorithm then updates the pointers in the popped tuple so they point to the newly copied tuples. Getting back to Figure 5.6, in the first step we copy the tuple whose second element is 42 to the back of the queue. The other pointer goes to a tuple that has already been copied, so we do not need to copy it again, but we do need to update the pointer to the new location. This can be accomplished by storing a forwarding pointer to the new location in the old tuple, back when we initially copied the tuple into the ToSpace. This completes one step of the algorithm. The algorithm continues in this way until the front of the queue is empty, that is, until the front catches up with the back.

5.2.2 Data Representation

The garbage collector places some requirements on the data representations used by our compiler. First, the garbage collector needs to distinguish between pointers and other kinds of data. There are several ways to accomplish this.

1. Attached a tag to each object that identifies what type of object it is [McCarthy 1960].

2. Store different types of objects in different regions [Steele 1977].

3. Use type information from the program to either generate type-specific code for collecting or to generate tables that can guide the collector [Appel 1989, Goldberg 1991, Diwan et al. 1992].
Figure 5.6: Depiction of the Cheney algorithm copying the live tuples.
Dynamically typed languages, such as Lisp, need to tag objects anyways, so option 1 is a natural choice for those languages. However, $R_3$ is a statically typed language, so it would be unfortunate to require tags on every object, especially small and pervasive objects like integers and Booleans. Option 3 is the best-performing choice for statically typed languages, but comes with a relatively high implementation complexity. To keep this chapter to a 2-week time budget, we recommend a combination of options 1 and 2, with separate strategies used for the stack and the heap.

Regarding the stack, we recommend using a separate stack for pointers [Siebert 2001; Henderson 2002; Baker et al. 2009], which we call a root stack (a.k.a. "shadow stack"). That is, when a local variable needs to be spilled and is of type (Vector $\text{type}_1 \ldots \text{type}_n$), then we put it on the root stack instead of the normal procedure call stack. Furthermore, we always spill vector-typed variables if they are live during a call to the collector, thereby ensuring that no pointers are in registers during a collection. Figure 5.7 reproduces the example from Figure 5.5 and contrasts it with the data layout using a root stack. The root stack contains the two pointers from the regular stack and also the pointer in the second register.

The problem of distinguishing between pointers and other kinds of data also arises inside of each tuple. We solve this problem by attaching a tag, an extra 64-bits, to each tuple. Figure 5.8 zooms in on the tags for two of the tuples in the example from Figure 5.5. Note that we have drawn the bits in a big-endian way, from right-to-left, with bit location 0 (the least
5.2. **GARBAGE COLLECTION**

Figure 5.8: Representation for tuples in the heap.

significant bit) on the far right, which corresponds to the directionality of the x86 shifting instructions `salq` (shift left) and `sarq` (shift right). Part of each tag is dedicated to specifying which elements of the tuple are pointers, the part labeled “pointer mask”. Within the pointer mask, a 1 bit indicates there is a pointer and a 0 bit indicates some other kind of data. The pointer mask starts at bit location 7. We have limited tuples to a maximum size of 50 elements, so we just need 50 bits for the pointer mask. The tag also contains two other pieces of information. The length of the tuple (number of elements) is stored in bits location 1 through 6. Finally, the bit at location 0 indicates whether the tuple has yet to be copied to the ToSpace. If the bit has value 1, then this tuple has not yet been copied. If the bit has value 0 then the entire tag is in fact a forwarding pointer. (The lower 3 bits of an pointer are always zero anyways because our tuples are 8-byte aligned.)

### 5.2.3 Implementation of the Garbage Collector

The implementation of the garbage collector needs to do a lot of bit-level data manipulation and we need to link it with our compiler-generated x86 code. Thus, we recommend implementing the garbage collector in C [Kernighan and Ritchie, 1988] and putting the code in the `runtime.c` file. Figure 5.9 shows the interface to the garbage collector. The `initialize` function creates the FromSpace, ToSpace, and root stack. The `initialize` function is meant to be called near the beginning of `main`, before the rest of the program executes. The `initialize` function puts the address of the beginning of the FromSpace into the global variable `free_ptr`. The global `fromspace_end` points to the address that is 1-past the last element of the FromSpace. (We use half-open intervals to represent chunks of memory [Dijkstra, 1982].) The
void initialize(uint64_t rootstack_size, uint64_t heap_size);
void collect(int64_t** rootstack_ptr, uint64_t bytes_requested);
int64_t* free_ptr;
int64_t* fromspace_begin;
int64_t* fromspace_end;
int64_t** rootstack_begin;

Figure 5.9: The compiler’s interface to the garbage collector.

rootstack_begin global points to the first element of the root stack.
As long as there is room left in the FromSpace, your generated code can
allocate tuples simply by moving the free_ptr forward. The amount of
room left in FromSpace is the difference between the fromspace_end and
the free_ptr. The collect function should be called when there is not
enough room left in the FromSpace for the next allocation. The collect
function takes a pointer to the current top of the root stack (one past the last
item that was pushed) and the number of bytes that need to be allocated.
The collect function performs the copying collection and leaves the heap
in a state such that the next allocation will succeed.

Exercise 19. In the file runtime.c you will find the implementation of
initialize and a partial implementation of collect. The collect func-
tion calls another function, cheney, to perform the actual copy, and that
function is left to the reader to implement. The following is the prototype
for cheney.

static void cheney(int64_t** rootstack_ptr);

The parameter rootstack_ptr is a pointer to the top of the rootstack (which
is an array of pointers). The cheney function also communicates with
collect through the global variables fromspace_begin and fromspace_end
mentioned in Figure 5.9 as well as the pointers for the ToSpace:

static int64_t* tospace_begin;
static int64_t* tospace_end;

The job of the cheney function is to copy all the live objects (reachable from
the root stack) into the ToSpace, update free_ptr to point to the next unused
spot in the ToSpace, update the root stack so that it points to the objects in
the ToSpace, and finally to swap the global pointers for the FromSpace and
ToSpace.

The introduction of garbage collection has a non-trivial impact on our
compiler passes. We introduce one new compiler pass called expose-allocation
and make non-trivial changes to type-check, flatten, select-instructions, allocate-registers, and print-x86. The following program will serve as our running example. It creates two tuples, one nested inside the other. Both tuples have length one. The example then accesses the element in the inner tuple tuple via two vector references.

\[(\text{vector-ref} (\text{vector-ref} (\text{vector} (42)) 0) 0)\]

Next we proceed to discuss the new expose-allocation pass.

### 5.3 Expose Allocation

The pass expose-allocation lowers the vector creation form into a conditional call to the collector followed by the allocation. We choose to place the expose-allocation pass before flatten because expose-allocation introduces new variables, which can be done locally with let, but let is gone after flatten. In the following, we show the transformation for the vector form into let-bindings for the initializing expressions, by a conditional collect, an allocate, and the initialization of the vector. (The \(\text{len}\) is the length of the vector and \(\text{bytes}\) is how many total bytes need to be allocated for the vector, which is 8 for the tag plus \(\text{len}\) times 8.)

\[
(\text{has-type} \left(\text{vector} \ e_0 \ldots e_{n-1}\right) \ \text{type})
\] \[\implies\]

\[
(\text{let} \ ([x_0 \ e_0]) \ldots (\text{let} \ ([x_{n-1} \ e_{n-1}]))
\]

\[
(\text{let} \ ([_ \ (\text{if} \ (< \ (+ \ (\text{global-value} \ \text{free_ptr}) \ \text{bytes}))
\]

\[
(\text{global-value} \ \text{fromspace_end}))
\]

\[
(\text{void})
\]

\[
(\text{collect} \ \text{bytes}))
\]

\[
(\text{let} \ ([v \ (\text{allocate} \ \text{len} \ \text{type})))
\]

\[
(\text{let} \ ([_ \ (\text{vector-set!} \ v \ 0 \ x_0)]) \ldots
\]

\[
(\text{let} \ ([_ \ (\text{vector-set!} \ v \ n-1 \ x_{n-1})))
\]

\[
(v) \ldots )))))) \ldots)
\]

(In the above, we suppressed all of the has-type forms in the output for the sake of readability.) The placement of the initializing expressions \(e_0, \ldots, e_{n-1}\) prior to the allocate and the sequence of vector-set!'s is important, as those expressions may trigger garbage collection and we do not want an allocated but uninitialized tuple to be present during a garbage collection.

The output of expose-allocation is a language that extends \(R_3\) with
(program ()
  (vector-ref (vector-ref
    (let ((vecinit48
      (let ((vecinit44 42))
        (let ((collectret46
          (if (<
            (+ (global-value free_ptr) 16)
            (global-value fromspace_end))
          (void)
            (collect 16))))))
      (let ((alloc43 (allocate 1 (Vector Integer))))
        (let ((initret45 (vector-set! alloc43 0 vecinit44)))
          alloc43)))))
    (let ((collectret50
      (if (< (+ (global-value free_ptr) 16)
        (global-value fromspace_end))
      (void)
        (collect 16)))))
      (let ((alloc47 (allocate 1 (Vector (Vector Integer))))
        (let ((initret49 (vector-set! alloc47 0 vecinit48)))
          alloc47)))))
  0)
0))

Figure 5.10: Output of the expose-allocation pass, minus all of the has-type forms.

the three new forms that we use above in the translation of vector.

\[ exp ::= \cdots | (collect \textit{int}) | (allocate \textit{int type}) | (\textit{global-value name}) \]

Figure 5.10 shows the output of the expose-allocation pass on our running example.

5.4 Explicate Control and the \(C_2\) language

The output of explicate-control is a program in the intermediate language \(C_2\), whose syntax is defined in Figure 5.11. The new forms of \(C_2\) include the allocate, vector-ref, and vector-set!, and global-value expressions and the collect statement. The explicate-control pass can treat these new forms much like the other forms.
5.5 **UNCOVER LOCALS**

Recall that the `uncover-locals` function collects all of the local variables so that it can store them in the `info` field of the `program` form. Also recall that we need to know the types of all the local variables for purposes of identifying the root set for the garbage collector. Thus, we change `uncover-locals` to collect not just the variables, but the variables and their types in the form of an association list. Thanks to the `has-type` forms, the types are readily available. Figure 5.12 lists the output of the `uncover-locals` pass on the running example.

| `arg`  | ::= | `int` | `var` | `#t` | `#f` |
|`cmp`   | ::= | `eq?` | `<` |
|`exp`   | ::= | `arg` | `(read)` | `(- arg)` | `(+ arg arg)` | `(not arg)` | `(cmp arg arg)` |
|        |    |       | `(allocate int type)` | `(vector-ref arg int)` |
|        |    |       | `(vector-set! arg int arg)` | `(global-value name)` | `(void)` |
|`stmt`  | ::= | `(assign var exp)` | `(return exp)` | `(collect int)` |
|`tail`  | ::= | `(return exp)` | `(seq stmt tail)` |
|        |    |       | `(goto label)` | `(if (cmp arg arg) (goto label) (goto label))` |
|`C_2`   | ::= | `(program info ((label . tail)+))` |

Figure 5.11: The $C_2$ language, extending $C_1$ (Figure 4.5) with vectors.

### 5.5 Uncover Locals

Recall that the `uncover-locals` function collects all of the local variables so that it can store them in the `info` field of the `program` form. Also recall that we need to know the types of all the local variables for purposes of identifying the root set for the garbage collector. Thus, we change `uncover-locals` to collect not just the variables, but the variables and their types in the form of an association list. Thanks to the `has-type` forms, the types are readily available. Figure 5.12 lists the output of the `uncover-locals` pass on the running example.
5. TUPLES AND GARBAGE COLLECTION

```
(program
  ((locals . ((tmp54 . Integer) (tmp51 . Integer) (tmp53 . Integer)
    (alloc43 . (Vector Integer)) (tmp55 . Integer)
    (initret45 . Void) (alloc47 . (Vector (Vector Integer)))
    (collectret46 . Void) (vecinit48 . (Vector Integer))
    (tmp52 . Integer) (tmp57 . (Vector Integer))
    (vecinit44 . Integer) (tmp56 . Integer) (initret49 . Void)
    (collectret50 . Void))))
  ((block63 . (seq (collect 16) (goto block61))))
  (block62 . (seq (assign collectret46 (void)) (goto block61))))
  (block61 . (seq (assign alloc43 (allocate 1 (Vector Integer)))
    (seq (assign initret45 (vector-set! alloc43 0 vecinit44))
    (seq (assign vecinit48 alloc43))
    (seq (assign tmp54 (global-value free_ptr))
    (seq (assign tmp55 (+ tmp54 16))
    (seq (assign tmp56 (global-value fromspace_end))
    (if (< tmp55 tmp56) (goto block59) (goto block60)))))))
  (block60 . (seq (collect 16) (goto block58))))
  (block59 . (seq (assign collectret50 (void)) (goto block58))))
  (block58 . (seq (assign alloc47 (allocate 1 (Vector (Vector Integer))))
    (seq (assign initret49 (vector-set! alloc47 0 vecinit48))
    (seq (assign tmp57 (vector-ref alloc47 0))
    (return (vector-ref tmp57 0))))))
  (start . (seq (assign vecinit44 42)
    (seq (assign tmp51 (global-value free_ptr))
    (seq (assign tmp52 (+ tmp51 16))
    (seq (assign tmp53 (global-value fromspace_end))
    (if (< tmp52 tmp53) (goto block62) (goto block63)))))))
```

Figure 5.12: Output of `uncover-locals` for the running example.
5.6. **SELECT INSTRUCTIONS**

5.6 Select Instructions

In this pass we generate x86 code for most of the new operations that were needed to compile tuples, including allocate, collect, vector-ref, vector-set!, and (void). We postpone global-value to print-x86.

The vector-ref and vector-set! forms translate into movq instructions with the appropriate deref. (The plus one is to get past the tag at the beginning of the tuple representation.)

\[
\begin{align*}
\text{(assign lhs (vector-ref vec n))} & \Rightarrow \\
& \text{(movq vec' (reg r11))} \\
& \text{(movq (deref r11 8(n + 1)) lhs)}
\end{align*}
\]

\[
\begin{align*}
\text{(assign lhs (vector-set! vec n arg))} & \Rightarrow \\
& \text{(movq vec' (reg r11))} \\
& \text{(movq arg' (deref r11 8(n + 1)))} \\
& \text{(movq (int 0) lhs)}
\end{align*}
\]

The vec' and arg' are obtained by recursively processing vec and arg. The move of vec' to register r11 ensures that offsets are only performed with register operands. This requires removing r11 from consideration by the register allocating.

We compile the allocate form to operations on the free_ptr, as shown below. The address in the free_ptr is the next free address in the FromSpace, so we move it into the lhs and then move it forward by enough space for the tuple being allocated, which is 8(len + 1) bytes because each element is 8 bytes (64 bits) and we use 8 bytes for the tag. Last but not least, we initialize the tag. Refer to Figure 5.8 to see how the tag is organized. We recommend using the Racket operations bitwise-ior and arithmetic-shift to compute the tag. The type annotation in the vector form is used to determine the pointer mask region of the tag.

\[
\begin{align*}
\text{(assign lhs (allocate len (Vector type...))}) & \Rightarrow \\
& \text{(movq (global-value free_ptr) lhs')} \\
& \text{(addq (int 8(len + 1)) (global-value free_ptr))} \\
& \text{(movq lhs' (reg r11))} \\
& \text{(movq (int tag) (deref r11 0))}
\end{align*}
\]

The collect form is compiled to a call to the collect function in the runtime. The arguments to collect are the top of the root stack and the number of bytes that need to be allocated. We shall use a dedicated register,
5. TUPLES AND GARBAGE COLLECTION

\[
\begin{align*}
\text{arg} &::= (\text{int int}) | (\text{reg register}) | (\text{deref register int}) \\
& \quad | (\text{byte-reg register}) | (\text{global-value name}) \\
\text{cc} &::= e | l | le | g | ge \\
\text{instr} &::= (\text{addq arg arg}) | (\text{subq arg arg}) | (\text{negq arg}) | (\text{movq arg arg}) \\
& \quad | (\text{callq label}) | (\text{pushq arg}) | (\text{popq arg}) | (\text{retq}) \\
& \quad | (\text{xorq arg arg}) | (\text{cmpq arg arg}) | (\text{setcc arg}) \\
& \quad | (\text{movzbq arg arg}) | (\text{jmp label}) | (\text{jmp-if cc label}) \\
& \quad | (\text{label label}) \\
x86_2 &::= (\text{program info (type type) instr}^+) \\
\end{align*}
\]

Figure 5.13: The x86\textsubscript{2} language (extends x86\textsubscript{1} of Figure 4.4).

\(r_{15}\), to store the pointer to the top of the root stack. So \(r_{15}\) is not available for use by the register allocator.

\[
\begin{align*}
(\text{collect bytes}) \\
\implies \\
(\text{movq (reg r15) (reg rdi)}) \\
(\text{movq bytes (reg rsi)}) \\
(\text{callq collect})
\end{align*}
\]

The syntax of the x86\textsubscript{2} language is defined in Figure 5.13. It differs from x86\textsubscript{1} just in the addition of the form for global variables. Figure 5.14 shows the output of the select-instructions pass on the running example.
5.6. SELECT INSTRUCTIONS

(program
  ((locals . ((tmp54 . Integer) (tmp51 . Integer) (tmp53 . Integer)
               (alloc43 . (Vector Integer)) (tmp55 . Integer)
               (initret45 . Void) (alloc47 . (Vector (Vector Integer)))
               (collectret46 . Void) (vecinit48 . (Vector Integer))
               (tmp52 . Integer) (tmp57 Vector Integer) (vecinit44 . Integer)
               (tmp56 . Integer) (initret49 . Void) (collectret50 . Void))))

  ((block63 . (block ()
               (movq (reg r15) (reg rdi))
               (movq (int 16) (reg rsi))
               (callq collect)
               (jmp block61)))

  (block62 . (block () (movq (int 0) (var collectret46)) (jmp block61)))

  (block61 . (block ()
               (movq (global-value free_ptr) (var alloc43))
               (addq (int 16) (global-value free_ptr))
               (movq (var alloc43) (reg r11))
               (movq (int 3) (deref r11 0))
               (movq (var alloc43) (reg r11))
               (movq (var vecinit44) (deref r11 8))
               (movq (int 0) (var initret45))
               (movq (var alloc43) (var vecinit48))
               (movq (global-value free_ptr) (var tmp54))
               (movq (var tmp54) (var tmp55))
               (addq (int 16) (var tmp55))
               (movq (global-value fromspace_end) (var tmp56))
               (cmpq (var tmp56) (var tmp55))
               (jmp-if 1 block59)
               (jmp block60)))

  (block60 . (block ()
               (movq (reg r15) (reg rdi))
               (movq (int 16) (reg rsi))
               (callq collect)
               (jmp block58))

  (block59 . (block ()
               (movq (int 0) (var collectret50))
               (jmp block58)))

  (block58 . (block ()
               (movq (global-value free_ptr) (var alloc47))
               (addq (int 16) (global-value free_ptr))
               (movq (var alloc47) (reg r11))
               (movq (int 131) (deref r11 0))
               (movq (var alloc47) (reg r11))
               (movq (var vecinit48) (deref r11 8))
               (movq (int 0) (var initret49))
               (movq (var alloc47) (reg r11))
               (movq (deref r11 8) (var tmp57))
               (movq (var tmp57) (reg r11))
               (movq (deref r11 8) (reg rax))
               (jmp conclusion)))))

  (start . (block ()
            (movq (int 42) (var vecinit44))
            (movq (global-value free_ptr) (var tmp51))
            (movq (var tmp51) (var tmp52))
            (addq (int 16) (var tmp52))
            (movq (global-value fromspace_end) (var tmp53))
            (cmpq (var tmp53) (var tmp52))
            (jmp-if 1 block62)
            (jmp block63))))

Figure 5.14: Output of the select-instructions pass.
5.7 Register Allocation

As discussed earlier in this chapter, the garbage collector needs to access all the pointers in the root set, that is, all variables that are vectors. It will be the responsibility of the register allocator to make sure that:

1. the root stack is used for spilling vector-typed variables, and
2. if a vector-typed variable is live during a call to the collector, it must be spilled to ensure it is visible to the collector.

The later responsibility can be handled during construction of the inference graph, by adding interference edges between the call-live vector-typed variables and all the callee-saved registers. (They already interfere with the caller-saved registers.) The type information for variables is in the program form, so we recommend adding another parameter to the build-interference function to communicate this association list.

The spilling of vector-typed variables to the root stack can be handled after graph coloring, when choosing how to assign the colors (integers) to registers and stack locations. The program output of this pass changes to also record the number of spills to the root stack.

5.8 Print x86

Figure 5.15 shows the output of the print-x86 pass on the running example. In the prelude and conclusion of the main function, we treat the root stack very much like the regular stack in that we move the root stack pointer (r15) to make room for all of the spills to the root stack, except that the root stack grows up instead of down. For the running example, there was just one spill so we increment r15 by 8 bytes. In the conclusion we decrement r15 by 8 bytes.

One issue that deserves special care is that there may be a call to collect prior to the initializing assignments for all the variables in the root stack. We do not want the garbage collector to accidentally think that some uninitialized variable is a pointer that needs to be followed. Thus, we zero-out all locations on the root stack in the prelude of main. In Figure 5.15, the instruction movq $0, (%r15) accomplishes this task. The garbage collector tests each root to see if it is null prior to dereferencing it.

Figure 5.16 gives an overview of all the passes needed for the compilation of $R_3$. 


Figure 5.15: Output of the print-x86 pass.
Figure 5.16: Diagram of the passes for $R_3$, a language with tuples.
6 Functions

This chapter studies the compilation of functions at the level of abstraction of the C language. This corresponds to a subset of Typed Racket in which only top-level function definitions are allowed. These kind of functions are an important stepping stone to implementing lexically-scoped functions in the form of lambda abstractions, which is the topic of Chapter 7.

6.1 The $R_4$ Language

The syntax for function definitions and function application is shown in Figure 6.1 where we define the $R_4$ language. Programs in $R_4$ start with zero or more function definitions. The function names from these definitions are in-scope for the entire program, including all other function definitions (so the ordering of function definitions does not matter). The syntax for function application does not include an explicit keyword, which is error prone when using match. To alleviate this problem, we change the syntax from $(exp \ exp^*)$ to $(app \ exp \ exp^*)$ during type checking.

Functions are first-class in the sense that a function pointer is data and can be stored in memory or passed as a parameter to another function. Thus, we introduce a function type, written

$$(\text{type}_1 \ \cdots \ \text{type}_n \rightarrow \text{type}_r)$$

for a function whose $n$ parameters have the types $\text{type}_1$ through $\text{type}_n$ and whose return type is $\text{type}_r$. The main limitation of these functions (with respect to Racket functions) is that they are not lexically scoped. That is, the only external entities that can be referenced from inside a function body are other globally-defined functions. The syntax of $R_4$ prevents functions from being nested inside each other.
Figure 6.1: Syntax of $R_4$, extending $R_3$ (Figure 5.2) with functions.

```
(type ::= Integer | Boolean | (Vector type*) | Void | (type* -> type)

cmp ::= eq? | < | <= | > | >=

def ::= (define (var [var: type]*): type exp)

(R_4 ::= (program info def*): exp)
```

Figure 6.2: Example of using functions in $R_4$.

The program in Figure 6.2 is a representative example of defining and using functions in $R_4$. We define a function `map-vec` that applies some other function `f` to both elements of a vector (a 2-tuple) and returns a new vector containing the results. We also define a function `add1` that does what its name suggests. The program then applies `map-vec` to `add1` and `(vector 0 41)`. The result is `(vector 1 42)`, from which we return the 42.

The definitional interpreter for $R_4$ is in Figure 6.3. The case for the `program` form is responsible for setting up the mutual recursion between the top-level function definitions. We use the classic backpatching approach that uses mutable variables and makes two passes over the function definitions [Kelsey et al., 1998]. In the first pass we set up the top-level environment using a mutable cons cell for each function definition. Note that the
6.2. FUNCTIONS IN X86

Lambda value for each function is incomplete; it does not yet include the environment. Once the top-level environment is constructed, we then iterate over it and update the lambda value’s to use the top-level environment.

6.2 Functions in x86

The x86 architecture provides a few features to support the implementation of functions. We have already seen that x86 provides labels so that one can refer to the location of an instruction, as is needed for jump instructions. Labels can also be used to mark the beginning of the instructions for a function. Going further, we can obtain the address of a label by using the leaq instruction and rip-relative addressing. For example, the following puts the address of the add1 label into the rbx register.

\[
\text{leaq add1(\%rip), \%rbx}
\]

In Section 2.2 we saw the use of the callq instruction for jumping to a function whose location is given by a label. Here we instead will be jumping to a function whose location is given by an address, that is, we need to make an indirect function call. The x86 syntax is to give the register name prefixed with an asterisk.

\[
\text{callq \ast\%rbx}
\]

6.2.1 Calling Conventions

The callq instruction provides partial support for implementing functions, but it does not handle (1) parameter passing, (2) saving and restoring frames on the procedure call stack, or (3) determining how registers are shared by different functions. These issues require coordination between the caller and the callee, which is often assembly code written by different programmers or generated by different compilers. As a result, people have developed conventions that govern how functions calls are performed. Here we shall use the same conventions used by the gcc compiler [Matz et al., 2013].

Regarding (1) parameter passing, the convention is to use the following six registers: rdi, rsi, rdx, rcx, r8, and r9, in that order. If there are more than six arguments, then the convention is to use space on the frame of the caller for the rest of the arguments. However, to ease the implementation of efficient tail calls (Section 6.2.2), we shall arrange to never have more than six arguments. The register rax is for the return value of the function.

Regarding (2) frames and the procedure call stack, the convention is that the stack grows down, with each function call using a chunk of space called
(define (interp-exp env)
  (lambda (e)
    (define recur (interp-exp env))
    (match e
      ...
      ['(,fun ,args ...)
       (define arg-vals (for/list ([e args]) (recur e)))
       (define fun-val (recur fun))
       (match fun-val
         [('(lambda (,xs ...) ,body ,fun-env)
           (define new-env (append (map cons xs arg-vals) fun-env))
           ((interp-exp new-env) body)]
         [else (error "interp-exp, expected function, not" fun-val)])
         [else (error 'interp-exp "unrecognized expression")])
    )))

(define (interp-def d)
  (match d
    ['(define (,f [,xs : ,ps] ...) : ,rt ,body)
     (mcons f '(lambda ,xs ,body ()))])
)

(define (interp-R4 p)
  (match p
    ['(program ,ds ... ,body)
     (let ([top-level (for/list ([d ds]) (interp-def d))])
      (for/list ([b top-level])
        (set-mcdr! b (match (mcdr b)
                           ['(lambda ,xs ,body ())
                            '(lambda ,xs ,body ,top-level)])
         ((interp-exp top-level) body)))]

Figure 6.3: Interpreter for the $R_4$ language.
a frame. The caller sets the stack pointer, register \( \text{rsp} \), to the last data item in its frame. The callee must not change anything in the caller’s frame, that is, anything that is at or above the stack pointer. The callee is free to use locations that are below the stack pointer.

Regarding (3) the sharing of registers between different functions, recall from Section 3.1 that the registers are divided into two groups, the caller-saved registers and the callee-saved registers. The caller should assume that all the caller-saved registers get overwritten with arbitrary values by the callee. Thus, the caller should either 1) not put values that are live across a call in caller-saved registers, or 2) save and restore values that are live across calls. We shall recommend option 1). On the flip side, if the callee wants to use a callee-saved register, the callee must save the contents of those registers on their stack frame and then put them back prior to returning to the caller. The base pointer, register \( \text{rbp} \), is used as a point-of-reference within a frame, so that each local variable can be accessed at a fixed offset from the base pointer. Figure 6.4 shows the layout of the caller and callee frames.
6. FUNCTIONS

6.2.2 Efficient Tail Calls

In general, the amount of stack space used by a program is determined by the longest chain of nested function calls. That is, if function $f_1$ calls $f_2$, $f_2$ calls $f_3$, . . . , and $f_{n-1}$ calls $f_n$, then the amount of stack space is bounded by $O(n)$. The depth $n$ can grow quite large in the case of recursive or mutually recursive functions. However, in some cases we can arrange to use only constant space, i.e. $O(1)$, instead of $O(n)$.

If a function call is the last action in a function body, then that call is said to be a tail call. In such situations, the frame of the caller is no longer needed, so we can pop the caller’s frame before making the tail call. With this approach, a recursive function that only makes tail calls will only use $O(1)$ stack space. Functional languages like Racket typically rely heavily on recursive functions, so they typically guarantee that all tail calls will be optimized in this way.

However, some care is needed with regards to argument passing in tail calls. As mentioned above, for arguments beyond the sixth, the convention is to use space in the caller’s frame for passing arguments. But here we’ve popped the caller’s frame and can no longer use it. Another alternative is to use space in the callee’s frame for passing arguments. However, this option is also problematic because the caller and callee’s frame overlap in memory. As we begin to copy the arguments from their sources in the caller’s frame, the target locations in the callee’s frame might overlap with the sources for later arguments! We solve this problem by not using the stack for parameter passing but instead use the heap, as we describe in the Section 6.5.

As mentioned above, for a tail call we pop the caller’s frame prior to making the tail call. The instructions for popping a frame are the instructions that we usually place in the conclusion of a function. Thus, we also need to place such code immediately before each tail call. These instructions include restoring the callee-saved registers, so it is good that the argument passing registers are all caller-saved registers.

One last note regarding which instruction to use to make the tail call. When the callee is finished, it should not return to the current function, but it should return to the function that called the current one. Thus, the return address that is already on the stack is the right one, and we should not use callq to make the tail call, as that would unnecessarily overwrite the return address. Instead we can simply use the jmp instruction. Like the indirect function call, we write an indirect jump with a register prefixed with an asterisk. We recommend using rax to hold the jump target because the preceeding “conclusion” overwrites just about everything else.
6.3 Shrink $R_4$

The shrink pass performs a couple minor modifications to the grammar to ease the later passes. This pass adds an empty info field to each function definition:

\[
\text{(define } (f \ [x_1 : \text{type}_1 \ldots] : \text{type}_r \ \text{exp}) \Rightarrow (\text{define } (f \ [x_1 : \text{type}_1 \ldots] : \text{type}_r \ () \ \text{exp})
\]

and introduces an explicit main function.

\[
\text{(program } \text{info } ds \ldots \text{exp}) \Rightarrow (\text{program } \text{info } ds' \ \text{mainDef})
\]

where mainDef is

\[
(\text{define } (\text{main}) : \text{Integer} () \ \text{exp}')
\]

6.4 Reveal Functions

Going forward, the syntax of $R_4$ is inconvenient for purposes of compilation because it conflates the use of function names and local variables. This is a problem because we need to compile the use of a function name differently than the use of a local variable; we need to use leaq to convert the function name (a label in x86) to an address in a register. Thus, it is a good idea to create a new pass that changes function references from just a symbol $f$ to (fun-ref $f$). A good name for this pass is reveal-functions and the output language, $F_1$, is defined in Figure 6.5.

Placing this pass after uniquify is a good idea, because it will make sure that there are no local variables and functions that share the same name. On the other hand, reveal-functions needs to come before the explicate-control pass because that pass will help us compile fun-ref into assignment statements.

6.5 Limit Functions

This pass transforms functions so that they have at most six parameters and transforms all function calls so that they pass at most six arguments. A simple strategy for imposing an argument limit of length $n$ is to take
all arguments \( i \) where \( i \geq n \) and pack them into a vector, making that subsequent vector the \( n \)th argument.

\[
(f \ x_1 \ldots \ x_n) \Rightarrow (f \ x_1 \ldots \ x_5 (\text{vector} \ x_6 \ldots \ x_n))
\]

In the body of the function, all occurrences of the \( i \)th argument in which \( i > 5 \) must be replaced with a \text{vector-ref}.

### 6.6 Remove Complex Operators and Operands

The primary decisions to make for this pass is whether to classify \text{fun-ref} and \text{app} as either simple or complex expressions. Recall that a simple expression will eventually end up as just an “immediate” argument of an x86 instruction. Function application will be translated to a sequence of instructions, so \text{app} must be classified as complex expression. Regarding \text{fun-ref}, as discussed above, the function label needs to be converted to an address using the \text{leaq} instruction. Thus, even though \text{fun-ref} seems rather simple, it needs to be classified as a complex expression so that we generate an assignment statement with a left-hand side that can serve as the target of the \text{leaq}.

### 6.7 Explicate Control and the \( C_3 \) language

Figure 6.6 defines the syntax for \( C_3 \), the output of \text{explicate-control}. The three mutually recursive functions for this pass, for assignment, tail, and predicate contexts, must all be updated with cases for \text{fun-ref} and \text{app}. In assignment and predicate contexts, \text{app} becomes \text{call}, whereas
### 6.8. Uncover Locals

The function for processing `tail` should be updated with a case for `tailcall`. We also recommend creating a new function for processing function definitions. Each function definition in \( C_3 \) has its own set of local variables, so the code for function definitions should be similar to the case for the `program` form in \( C_2 \).

#### 6.9 Select Instructions

The output of select instructions is a program in the x86\(_3\) language, whose syntax is defined in Figure 6.7.

An assignment of `fun-ref` becomes a `leaq` instruction as follows:

\[
(assign \ \text{lhs} \ (fun-ref \ f)) \quad \Rightarrow \quad (leaq \ (fun-ref \ f) \ \text{lhs})
\]

Regarding function definitions, we need to remove their parameters and
instead perform parameter passing in terms of the conventions discussed in Section 6.2. That is, the arguments will be in the argument passing registers, and inside the function we should generate a \texttt{movq} instruction for each parameter, to move the argument value from the appropriate register to a new local variable with the same name as the old parameter.

Next, consider the compilation of function calls, which have the following form upon input to \texttt{select-instructions}.

\begin{verbatim}
(assign lhs (call fun args . . .))
\end{verbatim}

In the mirror image of handling the parameters of function definitions, the arguments \texttt{args} need to be moved to the argument passing registers. Once the instructions for parameter passing have been generated, the function call itself can be performed with an indirect function call, for which I recommend creating the new instruction \texttt{indirect-callq}. Of course, the return value from the function is stored in \texttt{rax}, so it needs to be moved into the \texttt{lhs}.

\begin{verbatim}
(indirect-callq fun)
movq (reg rax) lhs
\end{verbatim}

Regarding tail calls, the parameter passing is the same as non-tail calls: generate instructions to move the arguments into to the argument passing registers. After that we need to pop the frame from the procedure call stack. However, we do not yet know how big the frame is; that gets determined during register allocation. So instead of generating those instructions here,
we invent a new instruction that means “pop the frame and then do an indirect jump”, which we name `tail-jmp`.

Recall that in Section 2.6 we recommended using the label `start` for the initial block of a program, and in Section 2.8 we recommended labelling the conclusion of the program with `conclusion`, so that `(return arg)` can be compiled to an assignment to `rax` followed by a jump to `conclusion`. With the addition of function definitions, we will have a starting block and conclusion for each function, but their labels need to be unique. We recommend prepending the function’s name to `start` and `conclusion`, respectively, to obtain unique labels. (Alternatively, one could `gensym` labels for the start and conclusion and store them in the `info` field of the function definition.)

### 6.10 Uncover Live

Inside `uncover-live`, when computing the W set (written variables) for an `indirect-callq` instruction, we recommend including all the caller-saved registers, which will have the affect of making sure that no caller-saved register actually needs to be saved.

### 6.11 Build Interference Graph

With the addition of function definitions, we compute an interference graph for each function (not just one for the whole program).

Recall that in Section 5.7 we discussed the need to spill vector-typed variables that are live during a call to the `collect`. With the addition of functions to our language, we need to revisit this issue. Many functions will perform allocation and therefore have calls to the collector inside of them. Thus, we should not only spill a vector-typed variable when it is live during a call to `collect`, but we should spill the variable if it is live during any function call. Thus, in the `build-interference` pass, we recommend adding interference edges between call-live vector-typed variables and the callee-saved registers (in addition to the usual addition of edges between call-live variables and the caller-saved registers).

### 6.12 Patch Instructions

In `patch-instructions`, you should deal with the x86 idiosyncrasy that the destination argument of `leaq` must be a register. Additionally, you should ensure that the argument of `tail-jmp` is `rax`, our reserved register—this
is to make code generation more convenient, because we will be trampling many registers before the tail call (as explained below).

### 6.13 Print x86

For the print-x86 pass, we recommend the following translations:

- \((\text{fun-ref } \text{label}) \Rightarrow \text{label}(\%\text{rip})\)
- \((\text{indirect-callq } \text{arg}) \Rightarrow \text{callq } *\text{arg}\)

Handling tail-jmp requires a bit more care. A straightforward translation of tail-jmp would be \(\text{jmp } *\text{arg}\), which is what we will want to do, but before the jump we need to pop the current frame. So we need to restore the state of the registers to the point they were at when the current function was called. This sequence of instructions is the same as the code for the conclusion of a function.

Note that your print-x86 pass needs to add the code for saving and restoring callee-saved registers, if you have not already implemented that. This is necessary when generating code for function definitions.

### 6.14 An Example Translation

Figure 6.8 shows an example translation of a simple function in R₄ to x86. The figure also includes the results of the explicate-control and select-instructions passes. We have omitted the has-type AST nodes for readability. Can you see any ways to improve the translation?

**Exercise 20.** Expand your compiler to handle R₄ as outlined in this section. Create 5 new programs that use functions, including examples that pass functions and return functions from other functions and including recursive functions. Test your compiler on these new programs and all of your previously created test programs.

Figure 6.9 gives an overview of the passes needed for the compilation of R₄.
6.14. AN EXAMPLE TRANSLATION

(program)
  (define (add86 [x87 : Integer] [y88 : Integer]) : Integer ()
    ((add86start .
      (return (addq (var x87) (var y88))))
    (define (main) : Integer ()
      ((mainstart .
        (seq (assign tmp89 (fun-ref add86))
          (tailcall tmp89 40 2))))))
⇒
  (define (add86) ((locals (x87 . Integer) (y88 . Integer))
    (num-params . 2))
    ((add86start .
      (block ()
        (movq (reg rcx) (var x87))
        (movq (reg rdx) (var y88))
        (movq (var x87) (reg rax))
        (movq (var y88) (reg rax))
        (jmp _add86conclusion)))))))

Figure 6.8: Example compilation of a simple function to x86.
Figure 6.9: Diagram of the passes for $R_4$, a language with functions.
Lexically Scoped Functions

This chapter studies lexically scoped functions as they appear in functional languages such as Racket. By lexical scoping we mean that a function’s body may refer to variables whose binding site is outside of the function, in an enclosing scope. Consider the example in Figure 7.1 featuring an anonymous function defined using the lambda form. The body of the lambda, refers to three variables: x, y, and z. The binding sites for x and y are outside of the lambda. Variable y is bound by the enclosing let and x is a parameter of f. The lambda is returned from the function f. Below the definition of f, we have two calls to f with different arguments for x, first 5 then 3. The functions returned from f are bound to variables g and h. Even though these two functions were created by the same lambda, they are really different functions because they use different values for x. Finally, we apply g to 11 (producing 20) and apply h to 15 (producing 22) so the result of this program is 42.

```
(define (f [x : Integer]) : (Integer -> Integer)
  (let ([y 4])
    (lambda: ([z : Integer]) : Integer
      (+ x (+ y z)))))

(let ([g (f 5)])
 (let ([h (f 3)])
  (+ (g 11) (h 15))))
```

Figure 7.1: Example of a lexically scoped function.
7. LEXICALLY SCOPED FUNCTIONS

Figure 7.2: Syntax of \( R_5 \), extending \( R_4 \) (Figure 6.1) with lambda.

7.1 The \( R_5 \) Language

The syntax for this language with anonymous functions and lexical scoping, \( R_5 \), is defined in Figure 7.2. It adds the lambda form to the grammar for \( R_4 \), which already has syntax for function application. In this chapter we shall describe how to compile \( R_5 \) back into \( R_4 \), compiling lexically-scoped functions into a combination of functions (as in \( R_4 \)) and tuples (as in \( R_3 \)).

To compile lexically-scoped functions to top-level function definitions, the compiler will need to provide special treatment to variable occurrences such as \( x \) and \( y \) in the body of the lambda of Figure 7.1, for the functions of \( R_4 \) may not refer to variables defined outside the function. To identify such variable occurrences, we review the standard notion of free variable.

**Definition 21.** A variable is free with respect to an expression \( e \) if the variable occurs inside \( e \) but does not have an enclosing binding in \( e \).

For example, the variables \( x, y, \) and \( z \) are all free with respect to the expression \( (+ x (+ y z)) \). On the other hand, only \( x \) and \( y \) are free with respect to the following expression because \( z \) is bound by the lambda.

\[
\text{(lambda: ([z : Integer]) : Integer}
\]
\[
(+ x (+ y z)))
\]

Once we have identified the free variables of a lambda, we need to arrange for some way to transport, at runtime, the values of those variables from the point where the lambda was created to the point where the lambda is applied. Referring again to Figure 7.1 the binding of \( x \) to 5 needs to be

```plaintext
type ::= Integer | Boolean | (Vector type\textsuperscript{*}) | Void | (type\textsuperscript{*} -> type)
exp ::= int | (read) | (- exp) | (+ exp exp) | (- exp exp) | var | (let ([var exp]) exp) | #t | #f | (and exp exp) | (or exp exp) | (not exp) | (eq? exp exp) | (if exp exp exp) | (vector exp\textsuperscript{*}) | (vector-ref exp int) | (vector-set! exp int exp) | (void) | (exp exp\textsuperscript{*}) | (lambda: ([var : type\textsuperscript{*}]: type exp)
def ::= (define (var [var : type\textsuperscript{*}]: type exp)
R_5 ::= (program def\textsuperscript{*} exp)
```
7.2. **INTERPRETING $R_5$**

Figure 7.3: Example closure representation for the lambda’s in Figure 7.1.

used in the application of $g$ to 11, but the binding of $x$ to 3 needs to be used in the application of $h$ to 15. An efficient solution to the problem, due to [Cardelli 1983], is to bundle into a vector the values of the free variables together with the function pointer for the lambda’s code, an arrangement called a flat closure (which we shorten to just “closure”). Fortunately, we have all the ingredients to make closures, Chapter 5 gave us vectors and Chapter 6 gave us function pointers. The function pointer shall reside at index 0 and the values for free variables will fill in the rest of the vector. Figure 7.3 depicts the two closures created by the two calls to $f$ in Figure 7.1. Because the two closures came from the same lambda, they share the same function pointer but differ in the values for the free variable $x$.

7.2 **Interpreting $R_5$**

Figure 7.4 shows the definitional interpreter for $R_5$. The clause for lambda saves the current environment inside the returned lambda. Then the clause for app uses the environment from the lambda, the lam-env, when interpreting the body of the lambda. The lam-env environment is extended with the mapping of parameters to argument values.

7.3 **Type Checking $R_5$**

Figure 7.5 shows how to type check the new lambda form. The body of the lambda is checked in an environment that includes the current environment (because it is lexically scoped) and also includes the lambda’s parameters. We require the body’s type to match the declared return type.
(define (interp-exp env)
  (lambda (e)
    (define recur (interp-exp env))
    (match e ...
      ['(lambda ([,xs : ,Ts] ...) : ,rT ,body)
        '(lambda ,xs ,body ,env)]
      ['(app ,fun ,args ...)
        (define fun-val ((interp-exp env) fun))
        (define arg-vals (map (interp-exp env) args))
        (match fun-val ...
          ['(lambda ([,xs ...]) ,body ,lam-env)
            (define new-env (append (map cons xs arg-vals) lam-env))
            ((interp-exp new-env) body)]
          [else (error "interp-exp,expected,function,not" fun-val)])
          [else (error 'interp-exp "unrecognized,expression")]])))
  )))

Figure 7.4: Interpreter for $R_5$.

(define (typecheck-R5 env)
  (lambda (e)
    (match e ...
      ['(lambda ([,xs : ,Ts] ...) : ,rT ,body)
        (define new-env (append (map cons xs Ts) env))
        (define bodyT ((typecheck-R5 new-env) body))
        (cond [(equal? rT bodyT)
          '(',@Ts -> ,rT)]
          [else (error "mismatch,in,return,type" bodyT rT)])
          [else (error "mismatch,in,return,type" bodyT rT)])
        )))
  )))

Figure 7.5: Type checking the lambda's in $R_5$. 
7.4 Closure Conversion

The compiling of lexically-scoped functions into top-level function definitions is accomplished in the pass `convert-to-closures` that comes after `reveal-functions` and before `limit-functions`.

As usual, we shall implement the pass as a recursive function over the AST. All of the action is in the clauses for `lambda` and `app`. We transform a `lambda` expression into an expression that creates a closure, that is, creates a vector whose first element is a function pointer and the rest of the elements are the free variables of the `lambda`. The `name` is a unique symbol generated to identify the function.

\[
(\text{lambda}: (ps \ldots) : rt \; body) \Rightarrow (\text{vector} \; \text{name} \; fvs \ldots)
\]

In addition to transforming each `lambda` into a `vector`, we must create a top-level function definition for each `lambda`, as shown below.

\[
(\text{define} \; (name \; [clos : (\text{Vector} \_ \; fvs \ldots)]) \; ps \ldots)
\]

\[
(\text{let} \; ([fvs_1 \; (\text{vector-ref} \; clos \; 1)])
  \ldots
  (\text{let} \; ([fvs_n \; (\text{vector-ref} \; clos \; n)])
  \text{body}' \ldots))
\]

The `clos` parameter refers to the closure. The `ps` parameters are the normal parameters of the `lambda`. The types `fvs` are the types of the free variables in the `lambda` and the underscore is a dummy type because it is rather difficult to give a type to the function in the closure’s type, and it does not matter. The sequence of `let` forms bind the free variables to their values obtained from the closure.

We transform function application into code that retrieves the function pointer from the closure and then calls the function, passing in the closure as the first argument. We bind `e'` to a temporary variable to avoid code duplication.

\[
(\text{app} \; e \; es \ldots) \Rightarrow (\text{let} \; ([tmp \; e'])
  (\text{app} \; (\text{vector-ref} \; tmp \; 0) \; tmp \; es'))
\]

There is also the question of what to do with top-level function definitions. To maintain a uniform translation of function application, we turn function references into closures.

\[
(\text{fun-ref} \; f) \Rightarrow (\text{vector} \; (\text{fun-ref} \; f))
\]

The top-level function definitions need to be updated as well to take an
extra closure parameter.

7.5 An Example Translation

Figure 7.6 shows the result of closure conversion for the example program demonstrating lexical scoping that we discussed at the beginning of this chapter.

```
(define (f [x : Integer]) : (Integer -> Integer)
  (let ([y 4])
    (lambda: ([z : Integer]) : Integer
      (+ x (+ y z))))
(let ([g (f 5)])
  (let ([h (f 3)])
    (+ (g 11) (h 15))))

⇓

(define (f (x : Integer)) : (Integer -> Integer)
  (let ((y 4))
    (lambda: ((z : Integer)) : Integer
      (+ x (+ y z))))
(let ((g (app (fun-ref f) 5)))
  (let ((h (app (fun-ref f) 3)))
    (+ (app g 11) (app h 15))))

⇓

(define (f (clos.1 : _) (x : Integer)) : (Integer -> Integer)
  (let ((y 4))
    (vector (fun-ref lam.1) x y)))
(define (lam.1 (clos.2 : _) (z : Integer)) : Integer
  (let ((x (vector-ref clos.2 1)))
    (let ((y (vector-ref clos.2 2)))
      (+ x (+ y z))))
(let ((g (let ((t.1 (vector (fun-ref f))))
            (app (vector-ref t.1 0) t.1 5)))
      (let ((h (let ((t.2 (vector (fun-ref f))))
                 (app (vector-ref t.2 0) t.2 3)))
        (+ (let ((t.3 g)) (app (vector-ref t.3 0) t.3 11))
          (let ((t.4 h)) (app (vector-ref t.4 0) t.4 15))))))
```

Figure 7.6: Example of closure conversion.
7.5. AN EXAMPLE TRANSLATION

Figure 7.7: Diagram of the passes for $R_5$, a language with lexically-scoped functions.
Figure 7.7 provides an overview of all the passes needed for the compilation of $R_5$. 

In this chapter we discuss the compilation of a dynamically typed language, named $R_7$, that is a subset of the Racket language. (Recall that in the previous chapters we have studied subsets of the Typed Racket language.) In dynamically typed languages, an expression may produce values of differing type. Consider the following example with a conditional expression that may return a Boolean or an integer depending on the input to the program.

$$(\text{not} \ (\text{if} \ (\text{eq} \ (\text{read}) \ 1) \ \#f \ 0))$$

Languages that allow expressions to produce different kinds of values are called polymorphic. There are many kinds of polymorphism, such as subtype polymorphism and parametric polymorphism [Cardelli and Wegner 1985]. The kind of polymorphism are talking about here does not have a special name, but it is the usual kind that arrises in dynamically typed languages.

Another characteristic of dynamically typed languages is that primitive operations, such as not, are often defined to operate on many different types of values. In fact, in Racket, the not operator produces a result for any kind of value: given #f it returns #t and given anything else it returns #f. Furthermore, even when primitive operations restrict their inputs to values of a certain type, this restriction is enforced at runtime instead of during compilation. For example, the following vector reference results in a run-time contract violation.

$$(\text{vector-ref} \ (\text{vector} \ 42) \ \#t)$$

The syntax of $R_7$, our subset of Racket, is defined in Figure 8.1. The definitional interpreter for $R_7$ is given in Figure 8.2.

Let us consider how we might compile $R_7$ to x86, thinking about the first example above. Our bit-level representation of the Boolean #f is zero and
8. DYNAMIC TYPING

Figure 8.1: Syntax of $R_7$, an untyped language (a subset of Racket).

$$
cmp ::= \text{eq?} | < | \leq | > | \geq
\exp ::= \text{int} | (\text{read}) | (- \exp) | (+ \exp \exp) | (- \exp \exp)
\quad | \\text{var} | (\text{let} ([\text{var} \exp]) \exp)
\quad | \#t | \#f | (\text{and} \exp \exp) | (\text{or} \exp \exp) | (\text{not} \exp)
\quad | (\text{cmp} \exp \exp) | (\text{if} \exp \exp \exp)
\quad | (\text{vector} \exp) | (\text{vector-ref} \exp \exp)
\quad | (\text{vector-set!} \exp \exp \exp) | (\text{void})
\quad | (\exp \exp) | (\text{lambda} (\text{var*}) \exp)
\quad | (\text{boolean?} \exp) | (\text{integer?} \exp)
\quad | (\text{vector?} \exp) | (\text{procedure?} \exp) | (\text{void?} \exp)
\def ::= (\text{define} (\text{var} \text{var*}) \exp)
\R_7 ::= (\text{program} \text{def*} \exp)
$$

similarly for the integer 0. However, (not #f) should produce #t whereas (not 0) should produce #f. Furthermore, the behavior of not, in general, cannot be determined at compile time, but depends on the runtime type of its input, as in the example above that depends on the result of (read).

The way around this problem is to include information about a value’s runtime type in the value itself, so that this information can be inspected by operators such as not. In particular, we shall steal the 3 right-most bits from our 64-bit values to encode the runtime type. We shall use 001 to identify integers, 100 for Booleans, 010 for vectors, 011 for procedures, and 101 for the void value. We shall refer to these 3 bits as the tag and we define the following auxiliary function.

$$
tagof(\text{Integer}) = 001
\tagof(\text{Boolean}) = 100
\tagof((\text{Vector}...)) = 010
\tagof((\text{Vectorof}...)) = 010
\tagof((...->...)) = 011
\tagof(\text{Void}) = 101
$$

(We shall say more about the new Vectorof type shortly.) This stealing of 3 bits comes at some price: our integers are reduced to ranging from $-2^{60}$ to $2^{60}$. The stealing does not adversely affect vectors and procedures because those values are addresses, and our addresses are 8-byte aligned so the rightmost 3 bits are unused, they are always 000. Thus, we do not lose
(define (get-tagged-type v) (match v [tagged ,v1 ,ty] ty))

(define (valid-op? op) (member op '(+ - and or not)))

(define (interp-r7 env)
  (lambda (ast)
    (define recur (interp-r7 env))
    (match ast
      [(? symbol?) (lookup ast env)]
      [(? integer?) '(inject ,ast Integer)]
      [#t '(inject #t Boolean)]
      [#f '(inject #f Boolean)]
      ['(read) '(inject ,(read-fixnum) Integer)]
      ['(lambda ,xs ...) ,body]
      [('lambda ,xs ,body ,env) (λ(map (lambda (x) 'Any) xs) -> Any)]
      ['(define ,f ,xs ...) ,body]
      (mcons f [('lambda ,xs ,body)])
      ['(program ,ds ... ,body)]
      (let ((top-level (for/list ((d ds)) (interp-r7 '())))
        (for/list (B top-level)
          (set-mcdr! b (match (mcdr b)
            ['(lambda ,xs ,body)
              (inject (lambda ,xs ,body ,top-level)
                (λ(map (lambda (x) 'Any) xs) -> Any))))))))
      ((interp-r7 top-level) body))
      ['(vector ,app recur elts) ...]
      (define tys (map get-tagged-type elts))
      [('inject ,apply vector els) (Vector ,@tys)]
      ['(vector-set! ,app recur v1) ,n ,app recur v2])
      (match v1
        ['(inject ,vec ,ty)
          (vector-set! vec n v2)]
        ['(inject (void) Void)])]
      ['(vector-ref ,app recur v) ,n]
      (match v [(inject ,vec ,ty) (vector-ref vec n)])
      ['(let ([,x ,app recur v]) ,body]
        (interp-r7 (cons (cons x v) env) body)]
      ['(op ,es ...) #:when (valid-op? op)
        (interp-r7-op op (for/list ((e esl) (recur e)))]
      ['(eq? ,app recur l) ,app recur r])
      (inject ,equal? 1 r Boolean])
      ['if ,app recur q ,t ,f]
      (match q
        ['(inject #f Boolean) (recur f)]
        [else (recur t)])]
      ['(app recur f-val) ,app recur vs] ...
      (match f-val
        ['(inject (lambda ,xs ... ,body ,lam-env) ,ty)
          (define new-env (append (map cons xs vs) lam-env))
          (interp-r7 new-env body)]
        [else (error "interp-r7,.,expected,.,function,.,not" f-val)])])

Figure 8.2: Interpreter for the R7 language. UPDATE ME - Jeremy
information by overwriting the rightmost 3 bits with the tag and we can simply zero-out the tag to recover the original address.

In some sense, these tagged values are a new kind of value. Indeed, we can extend our typed language with tagged values by adding a new type to classify them, called \texttt{Any}, and with operations for creating and using tagged values, yielding the \texttt{R6} language that we define in Section \ref{sec:right}. The \texttt{R6} language provides the fundamental support for polymorphism and runtime types that we need to support dynamic typing.

There is an interesting interaction between tagged values and garbage collection. A variable of type \texttt{Any} might refer to a vector and therefore it might be a root that needs to be inspected and copied during garbage collection. Thus, we need to treat variables of type \texttt{Any} in a similar way to variables of type \texttt{Vector} for purposes of register allocation, which we discuss in Section \ref{sec:register}. One concern is that, if a variable of type \texttt{Any} is spilled, it must be spilled to the root stack. But this means that the garbage collector needs to be able to differentiate between (1) plain old pointers to tuples, (2) a tagged value that points to a tuple, and (3) a tagged value that is not a tuple. We enable this differentiation by choosing not to use the tag 000. Instead, that bit pattern is reserved for identifying plain old pointers to tuples. On the other hand, if one of the first three bits is set, then we have a tagged value, and inspecting the tag can differentiation between vectors (010) and the other kinds of values.

We shall implement our untyped language \texttt{R7} by compiling it to \texttt{R6} (Section \ref{sec:compilation}), but first we describe the how to extend our compiler to handle the new features of \texttt{R6} (Sections \ref{sec:typed}, \ref{sec:operations}, and \ref{sec:register}).

### 8.1 The \texttt{R6} Language: Typed Racket + \texttt{Any}

The syntax of \texttt{R6} is defined in Figure \ref{fig:typed}. The \texttt{(inject e T)} form converts the value produced by expression \texttt{e} of type \texttt{T} into a tagged value. The \texttt{(project e T)} form converts the tagged value produced by expression \texttt{e} into a value of type \texttt{T} or else halts the program if the type tag is equivalent to \texttt{T}. We treat \texttt{(Vectorof Any)} as equivalent to \texttt{(Vector Any ...)}.

Note that in both \texttt{inject} and \texttt{project}, the type \texttt{T} is restricted to the flat types \texttt{ftype}, which simplifies the implementation and corresponds with what is needed for compiling untyped Racket. The type predicates, \texttt{(boolean? e)} etc., expect a tagged value and return \texttt{#t} if the tag corresponds to the predicate, and return \texttt{#f} otherwise. Selections from the type checker for \texttt{R6} are shown in Figure \ref{fig:checker} and the interpreter for \texttt{R6} is in Figure \ref{fig:interpreter}. 


8.1. THE $R_6$ LANGUAGE: TYPED RACKET + ANY

\[
\begin{align*}
\text{type} &::= \text{Integer} | \text{Boolean} | (\text{Vector } type') | (\text{Vectorof } type) | \text{Void} \\
&\quad | (type' \to type) | \text{Any} \\
\text{ftype} &::= \text{Integer} | \text{Boolean} | \text{Void} | (\text{Vectorof Any}) | (\text{Vector Any}^\ast) \\
&\quad | (\text{Any}^\ast \to \text{Any}) \\
\text{cmp} &::= \text{eq?} | \text{<} | \text{\leq} | \text{>} | \text{\geq} \\
\text{exp} &::= \text{int} | (\text{read}) | (\text{~ exp}) | (+ \text{ exp exp}) | (\text{~ exp exp}) \\
&\quad | \text{var} | (\text{let } ([\text{var}\exp]) \text{ exp}) \\
&\quad | \text{#t} | \text{#f} | (\text{and } \text{exp exp}) | (\text{or } \text{exp exp}) | (\text{not } \text{exp}) \\
&\quad | (\text{cmp } \text{exp exp}) | (\text{if } \text{exp exp exp}) \\
&\quad | (\text{vector } \text{exp}) | (\text{vector-ref } \text{exp int}) \\
&\quad | (\text{vector-set! } \text{exp int exp}) | (\text{void}) \\
&\quad | (\text{exp exp'}) | (\text{lambda: } ([\text{var}:\text{type}'\ast]:\text{type } \text{exp}) \\
&\quad | (\text{inject } \text{exp ftype}) | (\text{project } \text{exp ftype}) \\
&\quad | (\text{boolean? } \text{exp}) | (\text{integer? } \text{exp}) \\
&\quad | (\text{vector? } \text{exp}) | (\text{procedure? } \text{exp}) | (\text{void? } \text{exp}) \\
\text{def} &::= (\text{define } (\text{var } [\text{var}:\text{type}']':\text{type } \text{exp}) \\
\text{R}_6 &::= (\text{program } \text{def}^\ast \text{ exp})
\end{align*}
\]

Figure 8.3: Syntax of $R_6$, extending $R_5$ (Figure 7.2) with Any.
(define (flat-ty? ty) ...)

(define (typecheck-R6 env)
  (lambda (e)
    (define recur (typecheck-R6 env))
    (match e
      ['(inject ,e ,ty)
        (unless (flat-ty? ty)
          (error "may-only-inject\(,a\) value of flat type \(,\not\)-a" ty))
          (define-values (new-e e-ty) (recur e))
          (cond
            [(equal? e-ty ty)
              (values '(inject ,new-e ,ty) 'Any)]
            [else
              (error "inject \(,a\) to have \(,\not\)-a" e ty))]]
      ['(project ,e ,ty)
        (unless (flat-ty? ty)
          (error "may-only-project\(,a\) to \(,\not\)-a" ty))
          (define-values (new-e e-ty) (recur e))
          (cond
            [(equal? e-ty 'Any)
              (values '(project ,new-e ,ty) ty)]
            [else
              (error "project \(,\not\)-a to have \(,\not\)-a" e ty))]]
      ['(vector-ref ,e ,i)
        (define-values (new-e e-ty) (recur e))
        (match e-ty
          ['(Vector ,ts ...) ...]
          ['(Vectorof ,ty)
            (unless (exact-nonnegative-integer? i)
              (error "type-check "invalid-index\(,\not\)-a" i))
              (values '(vector-ref ,new-e ,i) ty)]
            [else (error "expected \(,\not\)-a in vector-ref,\(,\not\)-not e-ty))]]
          ...)
    ))
    Figure 8.4: Type checker for parts of the $R_6$ language.
(define primitives (set 'boolean? ...))

(define (interp-op op)
  (match op
    ['boolean? (lambda (v)
        (match v
          ['(tagged ,v1 Boolean) #t]
          [else #f]))]
...
))

;; Equivalence of flat types
(define (tyeq? t1 t2)
  (match '(,t1 ,t2)
    ['((Vectorof Any) (Vector ,t2s ...))
      (for/and ([t2 t2s]) (eq? t2 'Any))]
    ['((Vector ,t1s ...) (Vectorof Any))
      (for/and ([t1 t1s]) (eq? t1 'Any))]
    [else (equal? t1 t2)])

(define (interp-R6 env)
  (lambda (ast)
    (match ast
      ['(inject ,e ,t)
        '((tagged ,((interp-R6 env) e) ,t))]
      ['(project ,e ,t2)
        (define v ((interp-R6 env) e))
        (match v
          ['(tagged ,v1 ,t1)
            (cond [(tyeq? t1 t2)
              v1]
              [else
                (error "in_project,type_mismatch" t1 t2)])]
          [else
            (error "in_project,expected_tagged_value" v)])
      ...
    )))

Figure 8.5: Interpreter for $R_6$. 

8.2 Shrinking \( R_6 \)

In the shrink pass we recommend compiling \( \text{project} \) into an explicit if expression that uses three new operations: \( \text{tag-of-any} \), \( \text{value-of-any} \), and \( \text{exit} \). The \( \text{tag-of-any} \) operation retrieves the type tag from a tagged value of type \( \text{Any} \). The \( \text{value-of-any} \) retrieves the underlying value from a tagged value. Finally, the \( \text{exit} \) operation ends the execution of the program by invoking the operating system’s \( \text{exit} \) function. So the translation for \( \text{project} \) is as follows. (We have ommitted the \text{has-type} AST nodes to make this output more readable.)

\[
\text{(let } ([\text{tmp } e']) \\
\text{(project } e \text{ type}) \Rightarrow \\
\text{(if } (\text{eq? (tag-of-any tmp) tag}) \\
\text{(value-of-any tmp)} \\
\text{(exit)))}
\]

Similarly, we recommend translating the type predicates (\text{boolean?}, etc.) into uses of \( \text{tag-of-any} \) and \( \text{eq?} \).

8.3 Instruction Selection for \( R_6 \)

**Inject**  We recommend compiling an \text{inject} as follows if the type is \text{Integer} or \text{Boolean}. The \text{salq} instruction shifts the destination to the left by the number of bits specified its source argument (in this case 3, the length of the tag) and it preserves the sign of the integer. We use the \text{orq} instruction to combine the tag and the value to form the tagged value.

\[
\text{(assign } \text{lhs (inject } e \text{ T}) \\
\Rightarrow \\
\text{(movq } e' \text{ lhs'}) \\
\text{(salq (int 3) lhs'}) \\
\text{(orq (int tagof(T)) lhs'})
\]

The instruction selection for vectors and procedures is different because their is no need to shift them to the left. The rightmost 3 bits are already zeros as described above. So we just combine the value and the tag using \text{orq}.

\[
\text{(assign } \text{lhs (inject } e \text{ T}) \\
\Rightarrow \\
\text{(movq } e' \text{ lhs'}) \\
\text{(orq (int tagof(T)) lhs'})
\]

**Tag of Any**  Recall that the \( \text{tag-of-any} \) operation extracts the type tag from a value of type \( \text{Any} \). The type tag is the bottom three bits, so we obtain the tag by taking the bitwise-and of the value with 111 (7 in decimal).
8.4. REGISTER ALLOCATION FOR \( R_6 \)

\[
\text{Value of Any} \quad \text{Like inject, the instructions for value-of-any are different depending on whether the type } T \text{ is a pointer (vector or procedure) or not (Integer or Boolean). The following shows the instruction selection for Integer and Boolean. We produce an untagged value by shifting it to the right by 3 bits.}
\]

\[
\begin{align*}
\text{Value of Any} & \quad \text{Like inject, the instructions for value-of-any are different depending on whether the type } T \text{ is a pointer (vector or procedure) or not (Integer or Boolean). The following shows the instruction selection for Integer and Boolean. We produce an untagged value by shifting it to the right by 3 bits.}
\end{align*}
\]

\[
(\text{assign } lhs \ (\text{tag-of-any } e)) \quad \Rightarrow \quad (\text{movq } e' \ lhs') \\
(\text{andq } (\text{int 7}) \ lhs')
\]

\[
(\text{assign } lhs \ (\text{project } e \ T)) \quad \Rightarrow \quad (\text{movq } e' \ lhs') \\
(\text{sarq } (\text{int 3}) \ lhs')
\]

In the case for vectors and procedures, there is no need to shift. Instead we just need to zero-out the rightmost 3 bits. We accomplish this by creating the bit pattern \ldots0111 (7 in decimal) and apply bitwise-not to obtain \ldots1000 which we movq into the destination \( lhs \). We then generate andq with the tagged value to get the desired result.

\[
(\text{assign } lhs \ (\text{project } e \ T)) \quad \Rightarrow \quad (\text{movq } (\text{int } \ldots1000) \ lhs') \\
(\text{andq } e' \ lhs')
\]

8.4 Register Allocation for \( R_6 \)

As mentioned above, a variable of type \textit{Any} might refer to a vector. Thus, the register allocator for \( R_6 \) needs to treat variable of type \textit{Any} in the same way that it treats variables of type \textit{Vector} for purposes of garbage collection. In particular,

- If a variable of type \textit{Any} is live during a function call, then it must be spilled. One way to accomplish this is to augment the pass \texttt{build-interference} to mark all variables that are live after a \texttt{callq} as interfering with all the registers.

- If a variable of type \textit{Any} is spilled, it must be spilled to the root stack instead of the normal procedure call stack.

8.5 Compiling \( R_7 \) to \( R_6 \)

Figure 8.6 shows the compilation of many of the \( R_7 \) forms into \( R_6 \). An important invariant of this pass is that given a subexpression \( e \) of \( R_7 \), the pass will produce an expression \( e' \) of \( R_6 \) that has type \textit{Any}. For example,
<table>
<thead>
<tr>
<th>Expression</th>
<th>Compilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>#t</td>
<td>(inject #t Boolean)</td>
</tr>
<tr>
<td>(+ e₁ e₂)</td>
<td>(inject (+ (project e₁' Integer) (project e₂' Integer)) Integer)</td>
</tr>
<tr>
<td>(lambda (x₁...) e)</td>
<td>(inject (lambda: ([x₁:Any]...):Any e') (Any...Any -&gt; Any))</td>
</tr>
<tr>
<td>(app e₀ e₁...eₙ)</td>
<td>(app (project e₀' (Any...Any -&gt; Any)) e₁'...eₙ')</td>
</tr>
<tr>
<td>(vector-ref e₁ e₂)</td>
<td>(let ([tmp1 (project e₁' (Vectorof Any))]) (let ([tmp2 (project e₂' Integer)]) (vector-ref tmp1 tmp2)))</td>
</tr>
<tr>
<td>(if e₁ e₂ e₃)</td>
<td>(if (eq? e₁' (inject #f Boolean)) e₃' e₂')</td>
</tr>
<tr>
<td>(eq? e₁ e₂)</td>
<td>(inject (eq? e₁' e₂') Boolean)</td>
</tr>
</tbody>
</table>

Figure 8.6: Compiling R₇ to R₆.

The first row in Figure 8.6 shows the compilation of the Boolean #t, which must be injected to produce an expression of type Any. The second row of Figure 8.6, the compilation of addition, is representative of compilation for many operations: the arguments have type Any and must be projected to Integer before the addition can be performed.

The compilation of lambda (third row of Figure 8.6) shows what happens when we need to produce type annotations: we simply use Any. The compilation of if and eq? demonstrate how this pass has to account for some differences in behavior between R₇ and R₆. The R₇ language is more permissive than R₆ regarding what kind of values can be used in various places. For example, the condition of an if does not have to be a Boolean. For eq?, the arguments need not be of the same type (but in that case, the result will be #f).
Gradual Typing

This chapter will be based on the ideas of Sick and Taha [2006].
9. GRADUAL TYPING
Parametric Polymorphism

This chapter may be based on ideas from Cardelli 1984, Leroy 1992, Shao 1997, or Harper and Morrisett 1995.
11

High-level Optimization

This chapter will present a procedure inlining pass based on the algorithm of Waddell and Dybvig [1997].
Appendix

12.1 Interpreters

We provide several interpreters in the interp.rkt file. The interp-scheme function takes an AST in one of the Racket-like languages considered in this book ($R_1, R_2, \ldots$) and interprets the program, returning the result value. The interp-C function interprets an AST for a program in one of the C-like languages ($C_0, C_1, \ldots$), and the interp-x86 function interprets an AST for an x86 program.

12.2 Utility Functions

The utility function described in this section can be found in the utilities.rkt file.

The read-program function takes a file path and parses that file (it must be a Racket program) into an abstract syntax tree (as an S-expression) with a program AST at the top.

The assert function displays the error message msg if the Boolean bool is false.

(define (assert msg bool) ...)

The lookup function takes a key and an association list (a list of key-value pairs), and returns the first value that is associated with the given key, if there is one. If not, an error is triggered. The association list may contain both immutable pairs (built with cons) and mutable mapirs (built with mcons).

The map2 function ...
12.2.1 Testing

The `interp-tests` function takes a compiler name (a string), a description of the passes, an interpreter for the source language, a test family name (a string), and a list of test numbers, and runs the compiler passes and the interpreters to check whether the passes correct. The description of the passes is a list with one entry per pass. An entry is a list with three things: a string giving the name of the pass, the function that implements the pass (a translator from AST to AST), and a function that implements the interpreter (a function from AST to result value) for the language of the output of the pass. The interpreters from Appendix 12.1 make a good choice. The `interp-tests` function assumes that the subdirectory `tests` has a bunch of Scheme programs whose names all start with the family name, followed by an underscore and then the test number, ending in `.scm`. Also, for each Scheme program there is a file with the same number except that it ends with `.in` that provides the input for the Scheme program.

```scheme
(define (interp-tests name passes test-family test-nums) ...)
```

The compiler-tests function takes a compiler name (a string) a description of the passes (see the comment for `interp-tests`) a test family name (a string), and a list of test numbers (see the comment for `interp-tests`), and runs the compiler to generate x86 (a `.s` file) and then runs gcc to generate machine code. It runs the machine code and checks that the output is 42.

```scheme
(define (compiler-tests name passes test-family test-nums) ...)
```

The compile-file function takes a description of the compiler passes (see the comment for `interp-tests`) and returns a function that, given a program file name (a string ending in `.scm`), applies all of the passes and writes the output to a file whose name is the same as the program file name but with `.scm` replaced with `.s`.

```scheme
(define (compile-file passes)
  (lambda (prog-file-name) ...))
```

12.3 x86 Instruction Set Quick-Reference

Table 12.1 lists some x86 instructions and what they do. We write $A \rightarrow B$ to mean that the value of $A$ is written into location $B$. Address offsets are given in bytes. The instruction arguments $A, B, C$ can be immediate constants (such as $\$4$), registers (such as `%rax`), or memory references (such as
−4(%ebp)). Most x86 instructions only allow at most one memory reference per instruction. Other operands must be immediates or registers.
<table>
<thead>
<tr>
<th>Instruction</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>addq $A, B$</td>
<td>$A + B \rightarrow B$</td>
</tr>
<tr>
<td>negq $A$</td>
<td>$-A \rightarrow A$</td>
</tr>
<tr>
<td>subq $A, B$</td>
<td>$B - A \rightarrow B$</td>
</tr>
<tr>
<td>callq $L$</td>
<td>Pushes the return address and jumps to label $L$</td>
</tr>
<tr>
<td>callq $*A$</td>
<td>Calls the function at the address $A$.</td>
</tr>
<tr>
<td>retq</td>
<td>Pops the return address and jumps to it</td>
</tr>
<tr>
<td>popq $A$</td>
<td>$\ast rbp \rightarrow A; rbp + 8 \rightarrow rbp$</td>
</tr>
<tr>
<td>pushq $A$</td>
<td>$rbp - 8 \rightarrow rbp; A \rightarrow \ast rbp$</td>
</tr>
<tr>
<td>leaq $A, B$</td>
<td>$A \rightarrow B$ ($C$ must be a register)</td>
</tr>
<tr>
<td>cmpq $A, B$</td>
<td>Compare $A$ and $B$ and set the flag register</td>
</tr>
<tr>
<td>je $L$</td>
<td>Jump to label $L$ if the flag register matches the</td>
</tr>
<tr>
<td>jle $L$</td>
<td>condition code of the instruction, otherwise go to the</td>
</tr>
<tr>
<td>jge $L$</td>
<td>Jump to label $L$</td>
</tr>
<tr>
<td>jmp $L$</td>
<td></td>
</tr>
<tr>
<td>movq $A, B$</td>
<td>$A \rightarrow B$, where $A$ is a single-byte register (e.g., al or cl), $B$ is a 8-byte register, and the extra bytes of $B$ are set to zero.</td>
</tr>
<tr>
<td>movzbq $A, B$</td>
<td>$A \rightarrow B$, where $A$ is a single-byte register (e.g., al or cl), $B$ is a 8-byte register, and the extra bytes of $B$ are set to zero.</td>
</tr>
<tr>
<td>notq $A$</td>
<td>$\sim A \rightarrow A$ (bitwise complement)</td>
</tr>
<tr>
<td>orq $A, B$</td>
<td>$A</td>
</tr>
<tr>
<td>andq $A, B$</td>
<td>$A &amp; B \rightarrow B$ (bitwise-and)</td>
</tr>
<tr>
<td>salq $A, B$</td>
<td>$B \ll A \rightarrow B$ (arithmetic shift left, where $A$ is a constant)</td>
</tr>
<tr>
<td>sarq $A, B$</td>
<td>$B \gg A \rightarrow B$ (arithmetic shift right, where $A$ is a constant)</td>
</tr>
<tr>
<td>sete $A$</td>
<td>If the flag matches the condition code, then $1 \rightarrow A$, else</td>
</tr>
<tr>
<td>setl $A$</td>
<td>$0 \rightarrow A$. Refer to je above for the description of the</td>
</tr>
<tr>
<td>setle $A$</td>
<td>condition codes. $A$ must be a single byte register (e.g.,</td>
</tr>
<tr>
<td>setg $A$</td>
<td>al or cl).</td>
</tr>
<tr>
<td>setge $A$</td>
<td></td>
</tr>
</tbody>
</table>

Table 12.1: Quick-reference for the x86 instructions used in this book.
Bibliography


