In this assignment you will implement the Butterfly Subdivision algorithm shown in class. The provided code contains the half-edge data structure you will be using as your mesh data structure.

I. Data Structure

The half-edge data structure defines objects of the following types: Vertex, HalfEdge, and Face. If you have a vertex $v$, here’s what you can do with it:

- $v.getP os()$ returns the corresponding coordinates of the vertex; $v.setP os(x, y, z)$ changes the corresponding coordinates of the vertex
- $v.getEdge()$ returns the half-edge with $v$ as its origin; $v.setEdge(h)$ changes the half-edge with $v$ as its origin
- $v.getId()$ returns the index of $v$

Having a half-edge $h$, here’s what operations it supports:

- $h.getOrigin()$ returns the origin of $h$; $h.setOrigin(v)$ changes the origin of $h$
- $h.getFace()$ returns the face bounded by this half-edge; $h.setFace(f)$ changes the face bounded by that half-edge
- $h.getPrev()$ returns the previous half-edge in the face; $h.setPrev(h)$ changes the previous half-edge
- $h.getNext()$ returns the next half-edge in the face; $h.setNext(h)$ changes the next half-edge
- $h.getTwin()$ returns the twin half-edge; $h.setTwin(h)$ changes the twin half-edge

Having a face $f$, here’s what you can do with it:

- $f.getEdge()$ returns an arbitrary half-edge belonging to that face; $f.setEdge(h)$ changes the half-edge belonging to that face
- $f.vert(i)$ returns the $i$-th vertex of that face

II. Implementation of Butterfly Subdivision

Let’s split one iteration of mesh subdivision into two main stages: Stage A (Topological Subdivision) and Stage B (Vertex Placement). Start with implementing the first stage and only when you are sure it works correctly should you move on to the second one.

A. Topological subdivision.

At this first stage, given the initial mesh, represented as a half-edge data structure, you should create a new half-edge data structure, so that for every triangle in the input mesh you create four new triangles in the output mesh (Fig.).
Figure 1: Topological subdivision. Every triangle is replaced by four new triangles.

For this stage, use the midpoints of the old edges as the positions of the new vertices (this will help you debug your code). Do not change the positions of the old ones. The key requirement here is to split the triangle in four, properly updating all half-edges, vertices and faces.

Here’s one suggestion on how you can do it relatively easily. Create two functions, \textit{EdgeSplit}(\textit{he}) and \textit{CutACorner}(\textit{f}) with the following functionality:

\textit{EdgeSplit}(\textit{he}) will take input half-edge and split edge into two:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Diagram illustrating the process of subdividing a triangle.}
\end{figure}

Here’s the approximate pseudo-code for this function:

\textbf{function} \textit{SPLITEDGE}(\textit{HalfEdge} \textit{he}, \textit{Mesh} \textit{M})
\begin{itemize}
    \item add new vertex \textit{v} to \textit{M}
    \item mark \textit{v} as new \textit{⊿}
    \item store ‘isNew’ flag for every vertex
    \item add 2 new halfedges to \textit{M}
    \item set affected \textit{next}, \textit{prev}, \textit{twin}, \textit{origins}
    \item mark \textit{he}, \textit{he.twin} and the new halfedges as already split
    \item store ‘isSplit’ flag for every halfedge
\end{itemize}
\textbf{end function}

For each edge in the mesh, first run the \textit{EdgeSplit} function. After its completion, create the additional faces using the \textit{CutACorner} function which cuts the first corner it finds (unless the face is already a triangle):

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Diagram illustrating the process of cutting a corner.}
\end{figure}
The approximate pseudo-code will be along the lines of:

function CUTACORNER(Face f, Mesh M)
    Add new halfedge h to M
    Mark h as already split
    Update f.he() if necessary and add a new face to M
    Update all the necessary next, prev, twin, origins
end function

Once you have implemented these functions, the overall algorithm for one iteration of subdivision will be more or less straightforward:

function SUBDIVIDELOOP(Mesh M)
    mark all vertices in M as old
    mark all half-edges in M as not split
    while there is an half-edge he ∈ M that is not split yet do
        SPLITEDGE(he,M)
    end while
    while there is a non-triangle face f ∈ M do
        ▷ Think about how to check if face f is a triangle or not
        CUTACORNER(f,M)
    end while
end function

Once this is done, your topological subdivision part is finished. Test it by applying the subdivision step over and over - check that the code does not crash, all triangles remain visible and are exactly where you expect them to be.

B. Vertex Placement.

Now we need to compute the correct positions of all vertices. The formulas are different for the old and new vertices. Luckily, in the topological subdivision we set up the ‘new’ flag per every vertex exactly for this reason. This part should be relatively easy, for the formulas please refer to the lecture slides.

III. Running the template

Most browsers don’t allow access to local files, thus in order to run the template, you will need to set up a local server. A easy way to do it is via Python. Navigate to the working folder, and run one of commands listed below (depending on your version of Python). It will start a local server, then visit “http://localhost:8000/” within your browser and click on a7.html.

    python3 -m http.server — for Python 3
    python -m SimpleHTTPServer — for Python 2