CPSC 424
Subdivision Curves

Syllabus

Curves in 2D and 3D
- Implicit vs. Explicit vs. Parametric curves
- Bézier curves
- Continuity
- B-Splines
- Rational curves
- Subdivision curves

Properties of Curves and Surfaces
- Differential Geometry

Surfaces
Subdivision Curves

Represent smooth curve by approximating polyline

At the limit = curve

Each iteration

- Add new points (~double)
- Approximating - Move old points
- Interpolating – Keep old points

Bezier Subdivision

To create complex curve need to explicitly enforce continuity – not very useful
Corner Cutting

Corner Cutting
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Corner Cutting
Corner Cutting – Chaikin Algorithm

Limit – quadratic B-spline curve

Cubic B-Spline (corner cutting)
Cubic Corner Cutting

The 4-point scheme
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Why does it work?

\[ C(t) = at^3 + bt^2 + ct + d, \]
\[ C(0) = d \]

- A = C(-3) = -27a + 9b - 3c + d
- B = C(-1) = -a + b - c + d
- D = C(1) = a + b + c + d
- E = C(3) = 27a + 9b + 3c + d
- d = M + (M - M')/8,
  - with 2M = B + D and 2M' = A + E
- Vector from M to d = 1/8 of ||MM'||
Subdivision curves

Non interpolatory subdivision schemes
• Corner Cutting

Interpolatory subdivision schemes
• The 4-point scheme

Proving scheme works

Proving scheme works:
• Convergence
  – Will do on board & more details later
• Degree of continuity
• Affine invariance
  – As long as weights sum to 1
  – Proof on board
Clicker Question

The four-point subdivision scheme is
A. Approximating
B. Interpolatory
C. Both
D. Neither

Subdivision Matrix

On Board – for Chaikin subdivision

\[
\begin{pmatrix}
P_0^i \\
P_1^i \\
P_2^i \\
P_3^i \\
\end{pmatrix} = \begin{pmatrix}
0 & 1/4 & 3/4 & 0 & 0 \\
1/4 & 3/4 & 0 & 0 & 0 \\
0 & 3/4 & 1/4 & 0 & 0 \\
0 & 1/4 & 3/4 & 0 & 0 \\
0 & 0 & 3/4 & 1/4 & 0 \\
\end{pmatrix}^i \begin{pmatrix}
P_0^0 \\
P_1^0 \\
P_2^0 \\
P_3^0 \\
\end{pmatrix}
\]
Eigen Decomposition

Diagonalize subdivision matrix

• eigenvectors

• eigenvalues

• : vector of points in a neighborhood

(N+1)-vector of 2D/3D points

2D/3D vector

Eigenvalues

“Good” case:

• $\lambda_0 = 1$ & $|\lambda_i| < 1, i = 1, \ldots, n-1$

• can make $e_{\mathcal{H}}$ zero by moving control points (by affine invariance)
Clicker Question

*Does a scheme with a subdivision matrix*

\[
\begin{pmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{pmatrix}
\]

*converge?*

A. Yes

B. No

C. Not enough information

Subdominant Eigenvectors

*Next higher order terms*

- Assume \( \lambda_i > \lambda, i = 2, ..., n-1 \)
- move control points so that \( \varepsilon_{\mathbf{H}} \mathbf{I} \mathbf{H} \)

\[
\frac{f}{e} C^e \sim \mathbf{I} \varepsilon_{\mathbf{f}} f
\]

subdominant eigenvector

vanishes
Subdivision continuity

**Continuity of limit curve**

- Corner cutting (quadratic & cubic) – $C_{\text{inf}}$ nearly everywhere, $C_1/C_2$ at a finite number of points
  - B-Spline continuity

- Four-point scheme – $C_1$ everywhere