



CPSC 424

Subdivision Curves

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Syllabus

Curves in 2D and 3D

- Implicit vs. Explicit vs. Parametric curves
- Parametric Curves: Basis functions and Splines
- Splines: Bezier and Hermite
- Other bases
- Elementary differential geometry
- **Subdivision Curves**

Surfaces

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Subdivision Curves

*Represent smooth curve by approximating
polyline*

At the limit = curve

Each iteration

- Add new points (~double)
- Approximating - Move old points
- Interpolating – Keep old points

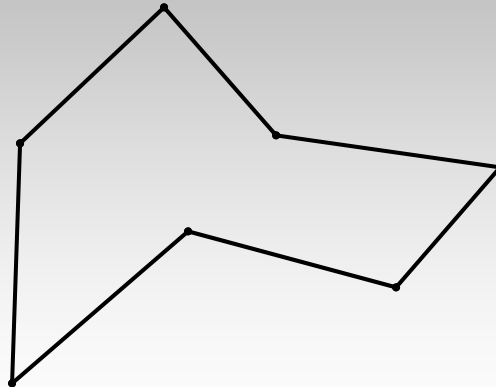
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Think how?

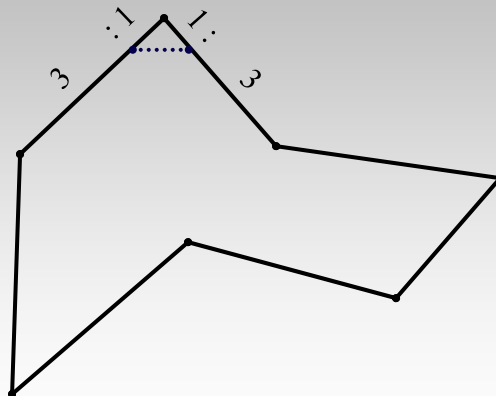
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Corner Cutting



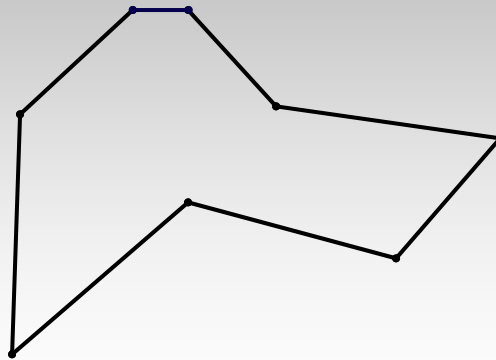
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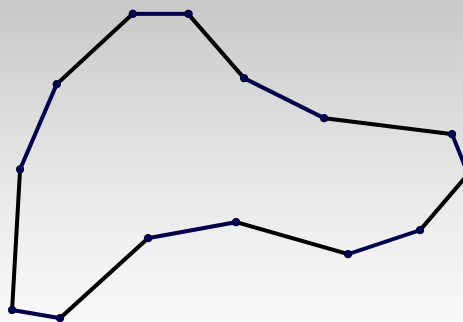
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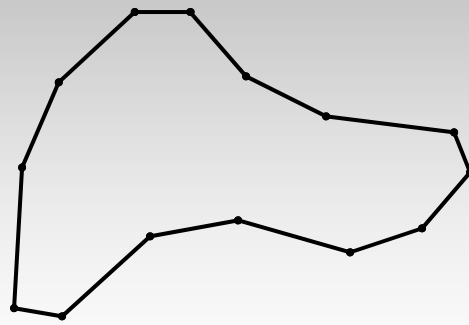
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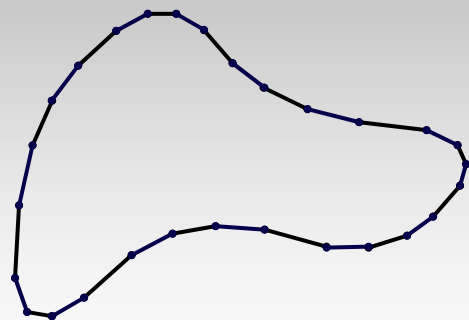
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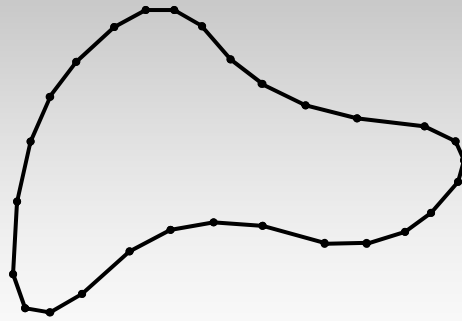
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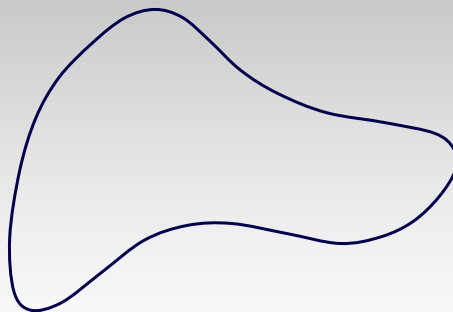
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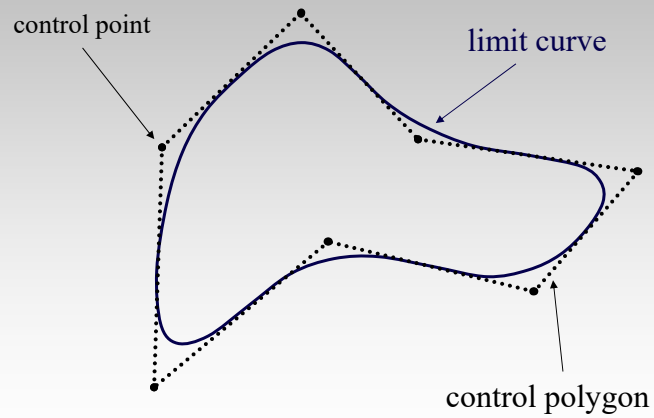
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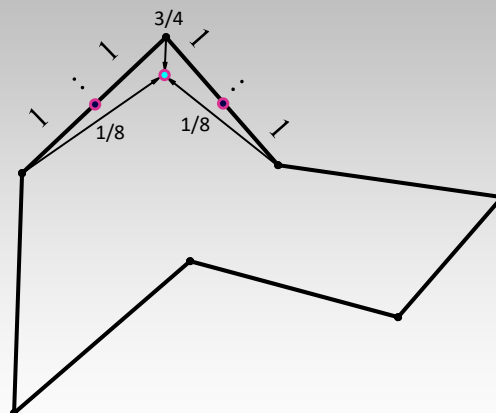
Corner Cutting – Chaikin Algorithm



Limit – quadratic B-spline curve

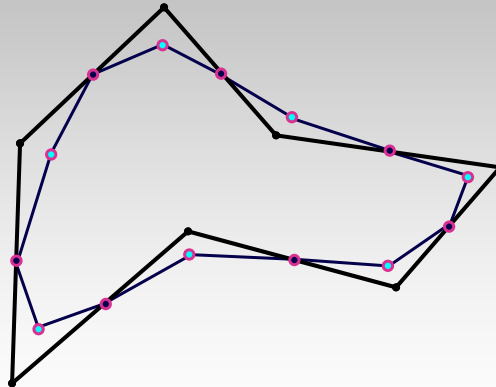
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Cubic B-Spline (corner cutting)



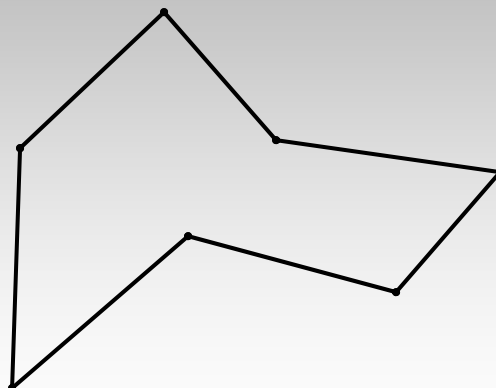
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Cubic Corner Cutting



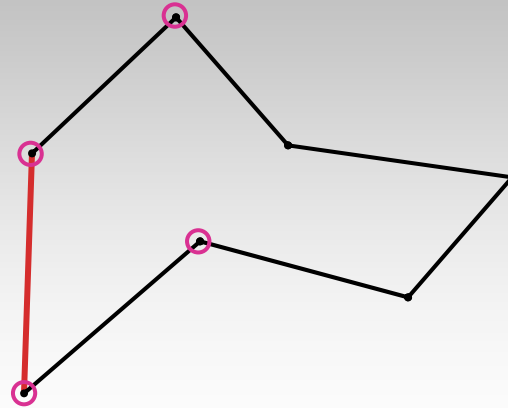
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The 4-point scheme



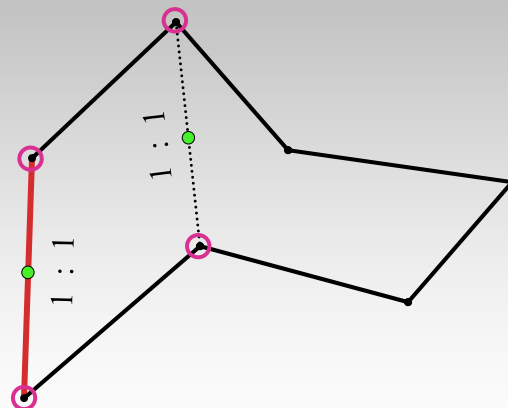
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The 4-point scheme



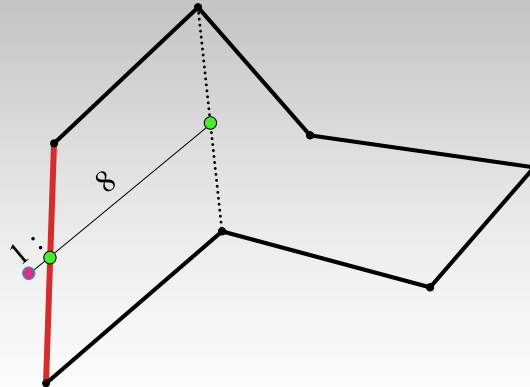
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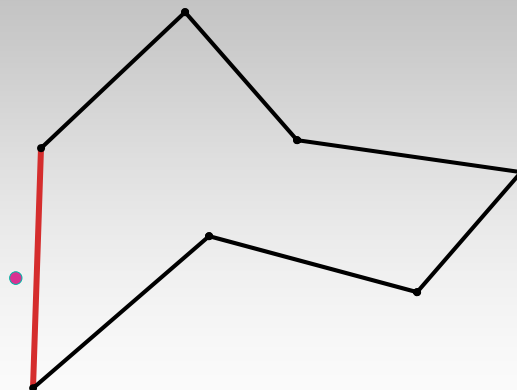
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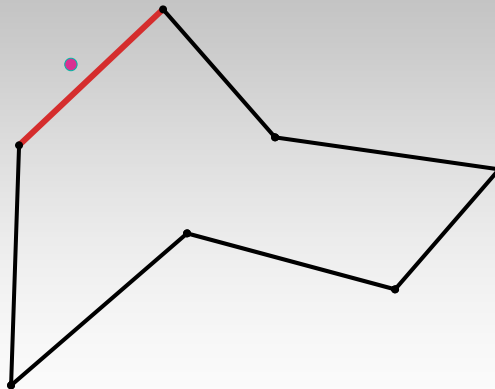
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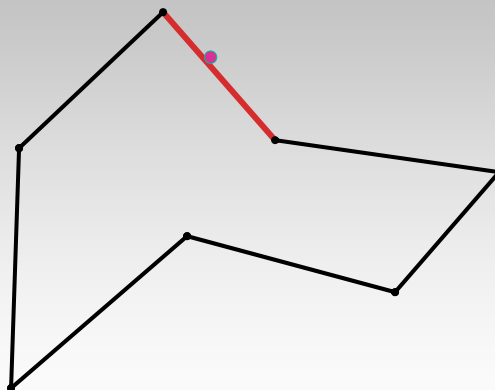
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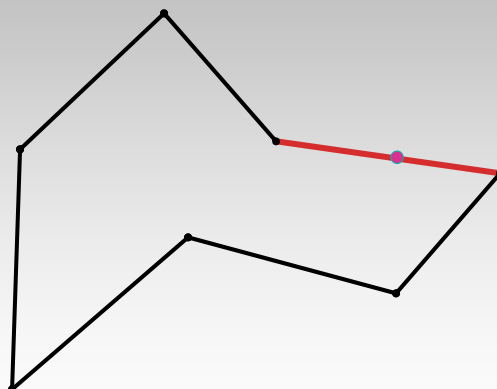
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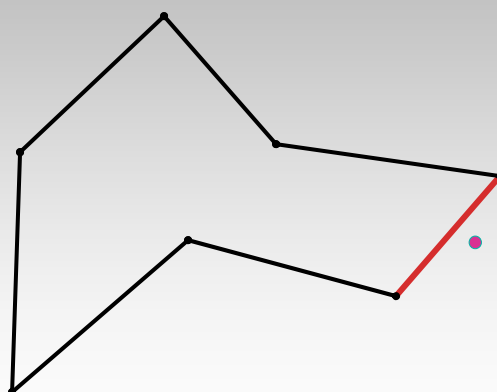
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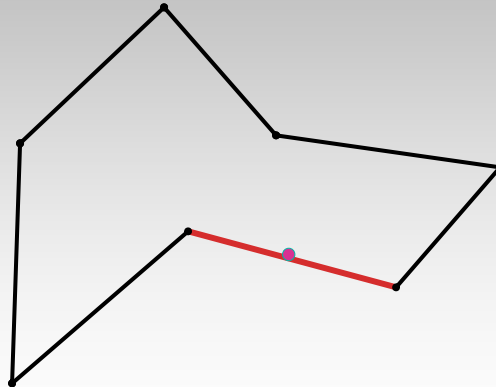
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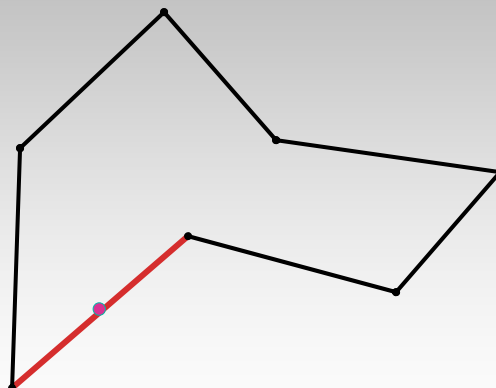
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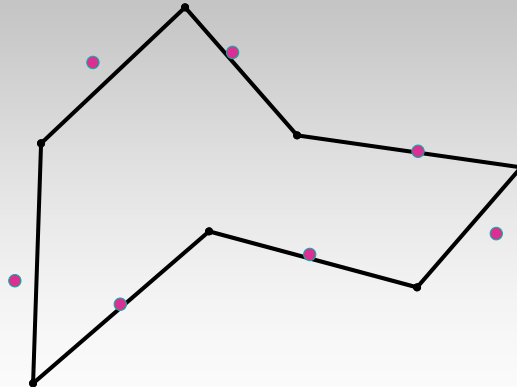
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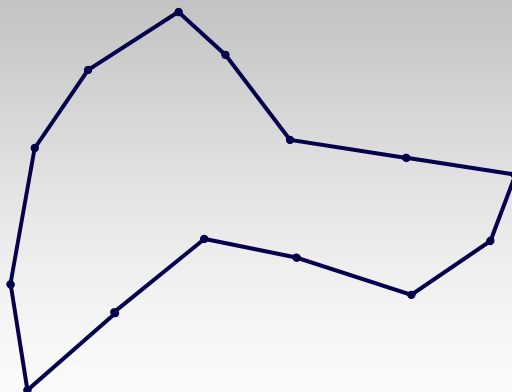
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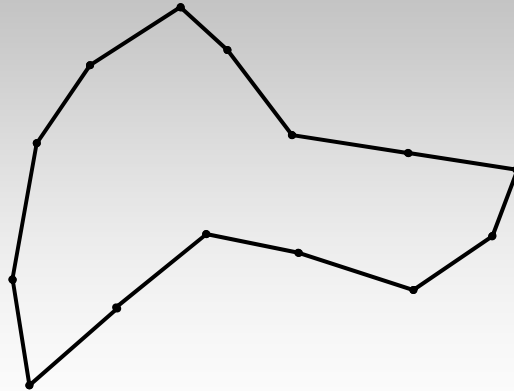
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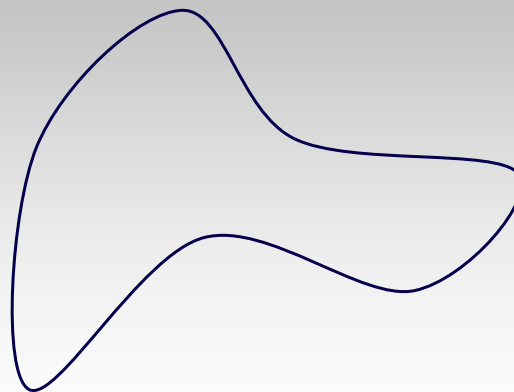
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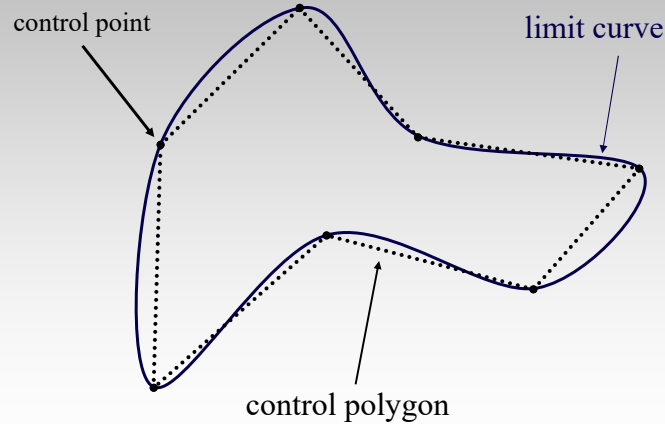


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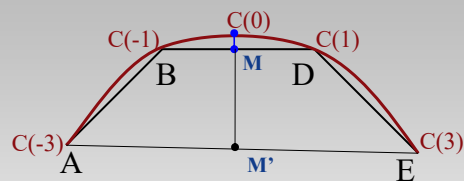
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Why does it work?

$$C(t) = at^3 + bt^2 + ct + d,$$

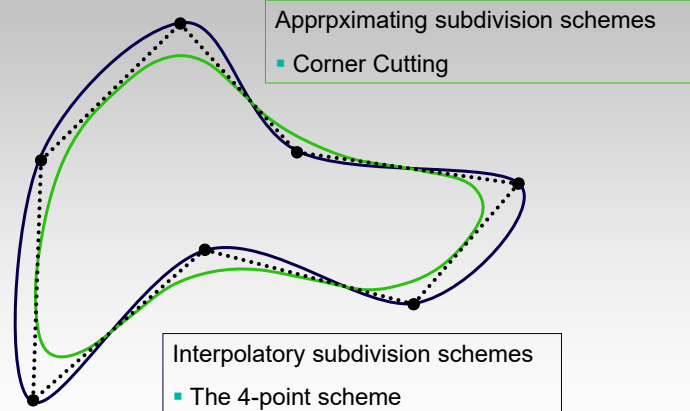
$$C(0) = d$$

- $A = C(-3) = -27a + 9b - 3c + d$
- $B = C(-1) = -a + b - c + d$
- $D = C(1) = a + b + c + d$
- $E = C(3) = 27a + 9b + 3c + d$
- $d = M + (M - M')/8,$
 - with $2M = B + D$ and $2M' = A + E$
- Vector from M to $d = 1/8$ of $\|MM'\|$



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Subdivision curves



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Proving scheme works

Proving scheme works:

- **Convergence**
 - *Will do on board & more details later*
- Degree of continuity
- Affine invariance
 - *As long as weights sum to 1*
 - *Proof on board*

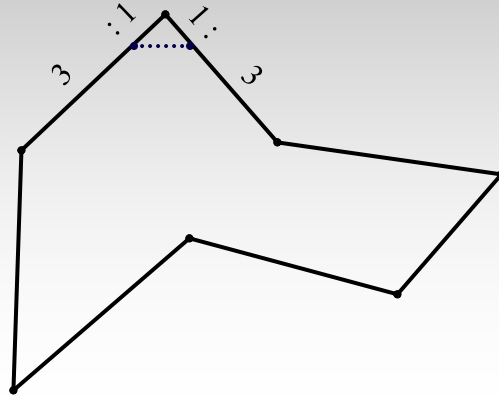
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Subdivision Matrix

On Board – for Chaikin subdivision

P_0 _____ P_1

$$\begin{pmatrix} P_i^0 \\ P_i^1 \end{pmatrix} = \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} P_{i-1}^0 \\ P_{i-1}^1 \end{pmatrix}$$



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Subdivision Matrix

Using a larger neighborhood

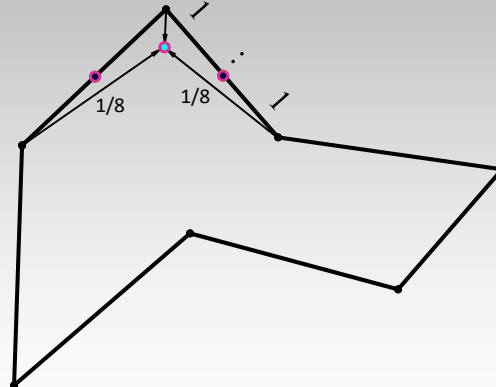


$$\begin{pmatrix} P_0^i \\ P_1^i \\ P_2^i \\ P_3^i \end{pmatrix} = \begin{pmatrix} 1/4 & 3/4 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 0 & 1/4 & 3/4 & 0 \\ 0 & 0 & 3/4 & 1/4 \end{pmatrix}^i \begin{pmatrix} P_0^0 \\ P_1^0 \\ P_2^0 \\ P_3^0 \end{pmatrix}$$

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Subdivision Matrix

On Board – for Cubic corner cutting



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Eigen Decomposition

Diagonalize subdivision matrix

- eigenvectors $x_i, i = 0..N$
- eigenvalues $\lambda_i, i = 0..N$
- p : vector of points in a neighborhood

$$p = \sum_i a_i x_i$$

(N+1)-vector of 3D points
3D vector

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Clicker Question

Does a scheme with a subdivision matrix

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

converge?

- A. Yes
- B. No
- C. Not enough information

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Eigenvectors

“Good” case:

- $\lambda_0 = 1$ & $|\lambda_i| < 1, i = 1, \dots, n-1$

$$S^m p = a_0 x_0 + \lambda_1^m a_1 x_1 + \lambda_2^m a_2 x_2 + \lambda_3^m a_3 x_3 + \dots$$

limit position

tangent vector

- can make a_0 zero by moving control points (by affine invariance)

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Subdominant Eigenvectors

Next higher order terms

- Assume $\lambda_1 > \lambda_i, i = 2, \dots, n-1$
- move control points so that $a_0 = 0$

$$\frac{1}{\lambda^m} S^m p = a_1 x_1 + \left(\frac{\lambda_2}{\lambda} \right)^m x_2 + \dots$$

subdominant eigenvector

vanishes

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Subdivision continuity

Continuity of limit curve

- Corner cutting (quadratic & cubic) – C_{inf} nearly everywhere, C_1/C_2 at a finite number of points
 - B-Spline continuity
- Four-point scheme – C_1 everywhere

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