CPSC 424
Meshes

Syllabus

Curves in 2D and 3D
Properties of Curves and Surfaces
Surfaces
- Tensor-product surfaces
- Bézier triangles
- Differential Geometry
- Polygonal meshes
- Subdivision Surfaces
Meshes

Standard Graph Definitions

\[ G = (V, E) \]
\[ V = \text{vertices} = \{A, B, C, D, E, F, G, H, I, J, K, L\} \]
\[ E = \text{edges} = \{(A, B), (B, C), (C, D), (D, E), (E, F), (F, G), (G, H), (H, A), (A, J), (A, G), (B, J), (K, F), (C, L), (C, I), (D, I), (D, F), (F, I), (G, K), (J, L), (J, K), (K, L), (L, I)\} \]

**Vertex degree (valence)** = number of edges incident on vertex

\[ \text{deg}(J) = 4, \text{deg}(H) = 2 \]

**Face**: cycle of vertices/edges which cannot be shortened

\[ F = \text{faces} = \{(A, H, G), (A, J, K, G), (B, A, J), (B, C, L, J), (C, I, L), (C, D, I), (D, E, F), (D, I, F), (L, I, F, K), (L, J, K), (K, F, G)\} \]
Connectivity

Graph is **connected** if there is a path of edges connecting every two vertices.

Graph $G' = \langle V', E' \rangle$ is a **subgraph** of graph $G = \langle V, E \rangle$ if $V'$ is a subset of $V$ and $E'$ is the subset of $E$ incident on $V'$.

**Connected component** of a graph: maximal connected subgraph.

How do we find if a graph is connected?
Graph Embedding

Graph is **embedded** in $\mathbb{R}^d$ if each vertex is assigned a position in $\mathbb{R}^d$ and the edges are represented by straight lines.

Embedding in $\mathbb{R}^2$

Embedding in $\mathbb{R}^3$

Meshes

**Mesh**: graph embedded in $\mathbb{R}^3$

**Boundary edge**: adjacent to exactly one face

**Regular edge**: adjacent to exactly two faces

**Singular edge**: adjacent to more than two faces

**Closed mesh**: mesh with no boundary edges

**Manifold** mesh: mesh with no singular edges

Non-Manifold  Closed Manifold  Open Manifold
Meshes

Triangular mesh – mesh all of whose faces have three vertices

We will discuss connected (3-connected) manifold triangular meshes (closed or open)

Clicker question

Is this graph connected? Is it manifold?

A. Yes & Yes
B. Yes & No
C. No & Yes
D. No & No
E. Not enough information
Clicker question

*What if I tell you that the ‘red’ edge is part of 3 faces?*

*Is this graph connected? Is it manifold?*

A. Yes & Yes  
B. Yes & No  
C. No & Yes  
D. No & No  
E. Not enough information

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Topology

Genus of graph: half of maximal number of closed paths that do *not* disconnect the graph (number of “holes”)

- Genus(sphere) = 0
- Genus(torus) = 1
Topology Quiz

What is the genus of a teapot?

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5

Topology Quiz

What is the genus of this shape?

A. 0
B. 2
C. 4
D. 6
E. None of the above
Topology

Euler-Poincare Formula

\[ v + f - e = 2(c - g) - b \]

- \( v \) = number of vertices
- \( f \) = number of faces
- \( e \) = number of edges
- \( c \) = number of connected components
- \( g \) = genus
- \( b \) = number of boundaries

How to find number of boundaries?
How to find number of connected components?

Topology

Euler-Poincare Formula

\[ v + f - e = 2(c - g) - b \]

- \( v \) = \# vertices
- \( f \) = \# faces
- \( e \) = \# edges
- \( c \) = \# conn. comp
- \( g \) = genus
- \( b \) = \# boundaries
Theorem: Average vertex degree in closed manifold triangle mesh is ~6

Proof: In such a mesh, $f = 2e/3$
By Euler's formula: $v + 2e/3 - e = 2 - 2g$

hence $e = 3(v-2+2g)$ and $f = 2(v-2+2g)$

So Average(deg) = $2e/v = 6(v-2+2g)/v$

~ 6 for large $v$

Corollary: Only toroidal (g=1) closed manifold triangle mesh can be regular (all vertex degrees are 6)

Proof: In regular mesh average degree is exactly 6
Can happen only if g=1
Exercises

**Theorem:** In closed manifold mesh: 
\[ 2e \geq 3f \] (equality for triangle mesh), 
\[ 2e \geq 3v \]

**Corollary:** No closed manifold triangle mesh can have 7 edges

**Corollary:** 
\[ 2f - 4 \geq v \]

Orientability

**Orientation** of a face is clockwise or anticlockwise order in which its vertices and edges are listed.

This defines the direction of face **normal**

Straight line graph is **orientable** if orientations of its faces can be chosen so that each edge is oriented in both directions
Moebius & Klein

Moebius strip or Klein bottle - not orientable