CPSC 424 - Tutorial 1

September 17th, 2026

TAs: Daniel Cui (dwcui@cs.ubc.ca), Oliver Oxford (oliox@cs.ubc.ca)

Mandatory Attendance

We will be calling students out at random **during tutorials** to do face-to-face assignment grading.

If you need to miss a tutorial for a valid reason, please message us on Piazza!

Assignment Submission

Option 1: Handin

See Piazza post for details

Option 2: Physical Hand-in

Hand in during a lecture OR leave your printed, submitted assignment in Prof. Alla Sheffer's CS department mailbox

Please let us know beforehand (Piazza) if you plan on using Option 2

Curve Representations

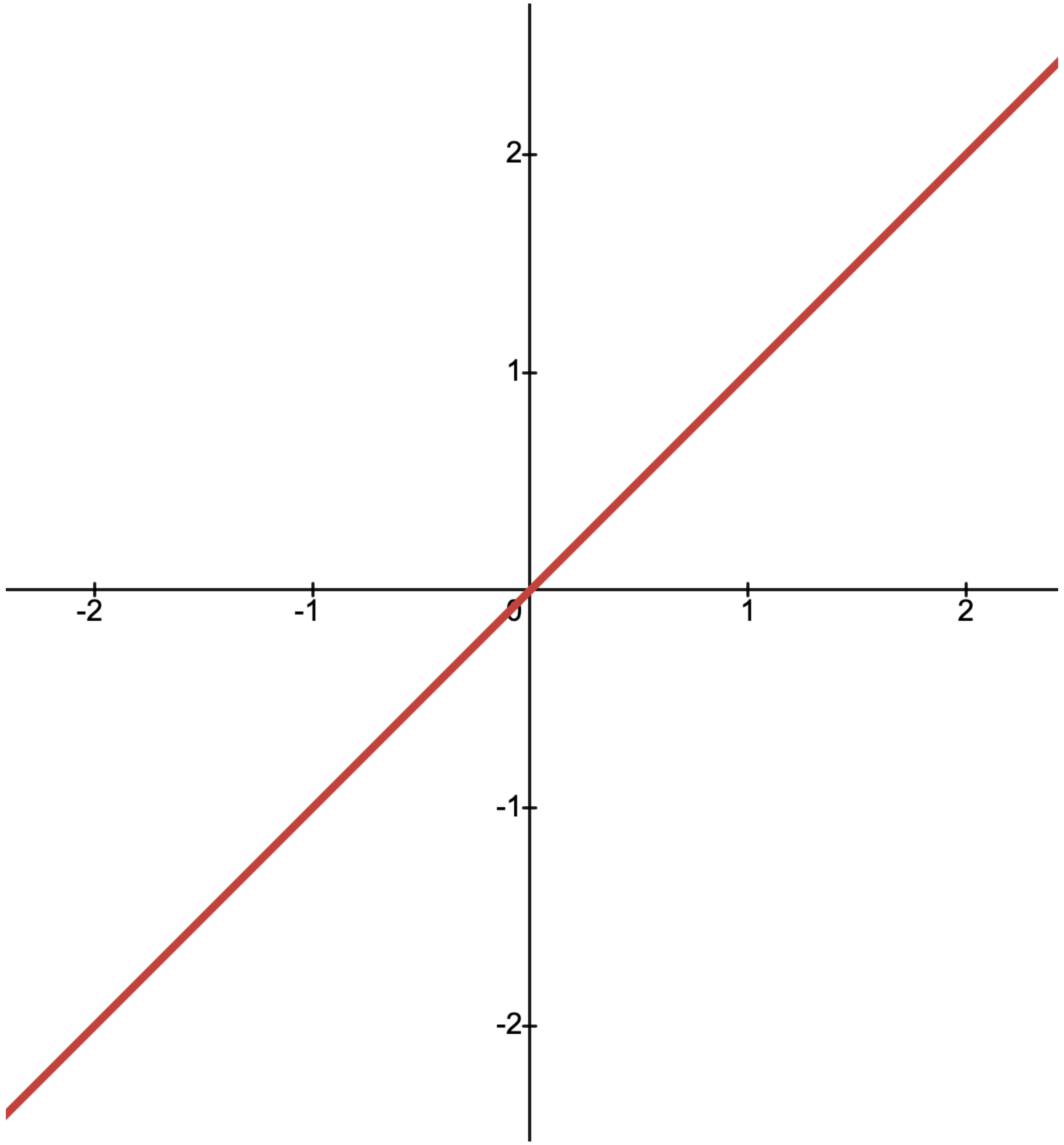
	2D	3D
Explicit	$y = F(x)$	$z = F(x, y)$
Implicit	$F(x, y) = 0$	$F(x, y, z) = 0$
Parametric	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix}$	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} F_x(t) \\ F_y(t) \\ F_z(t) \end{pmatrix}$ 3D Curve
		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} F_x(s, t) \\ F_y(s, t) \\ F_z(s, t) \end{pmatrix}$ 3D Surface

Curve Representations

Explicit

Implicit

Parametric



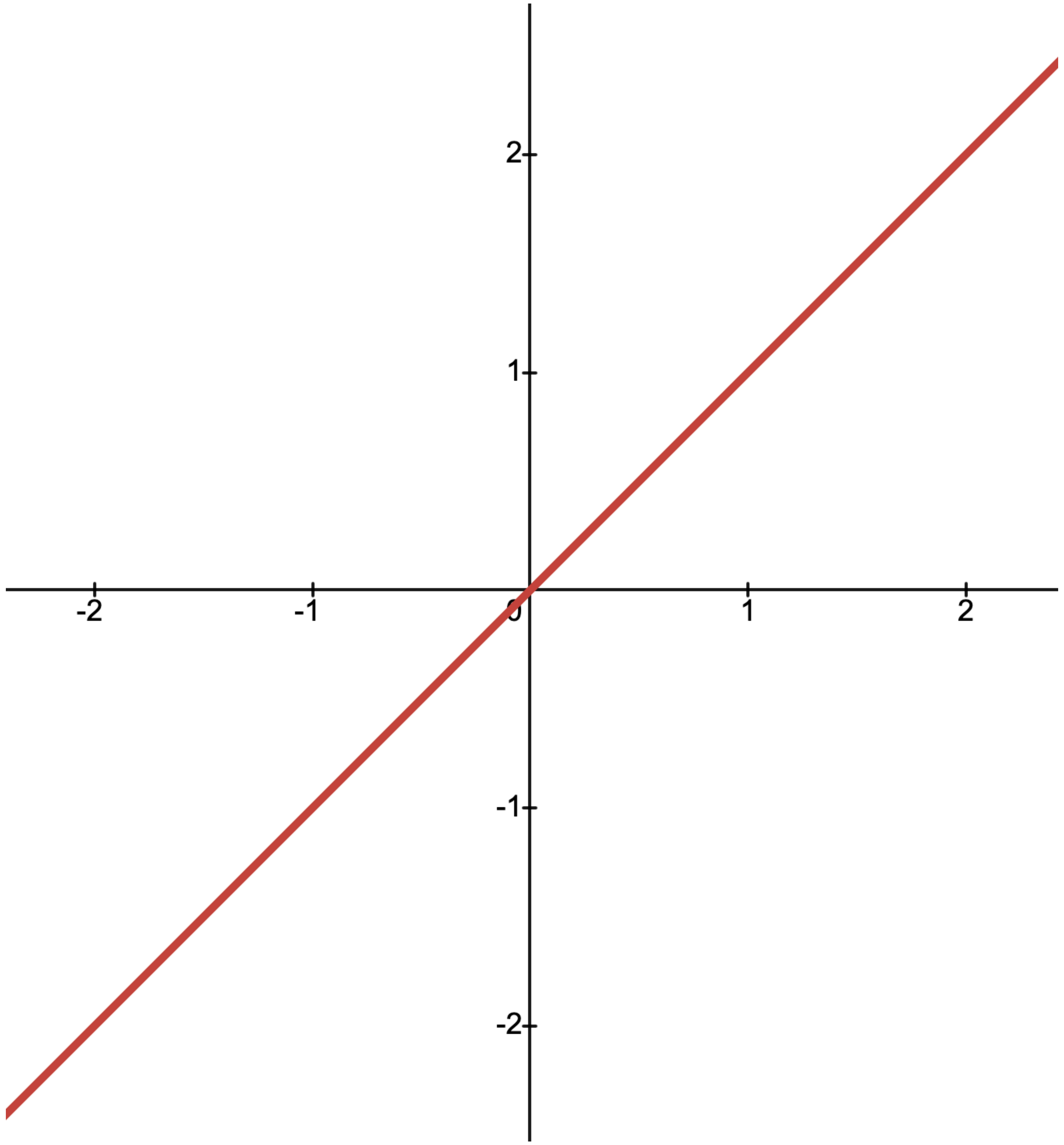
Curve Representations

Explicit

$$y = x$$

Implicit

Parametric



Curve Representations

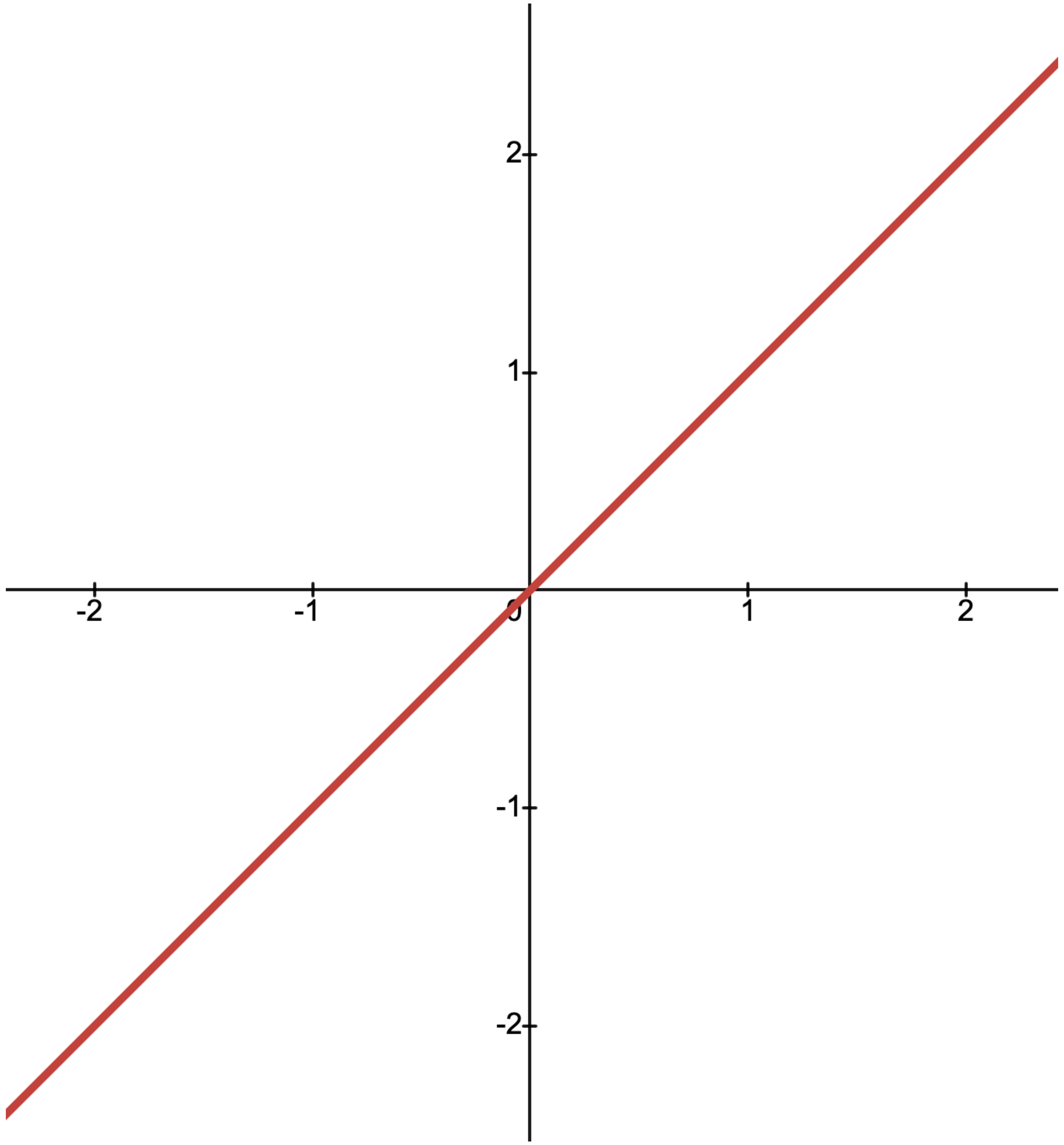
Explicit

$$y = x$$

Implicit

$$0 = x - y$$

Parametric



Curve Representations

Explicit

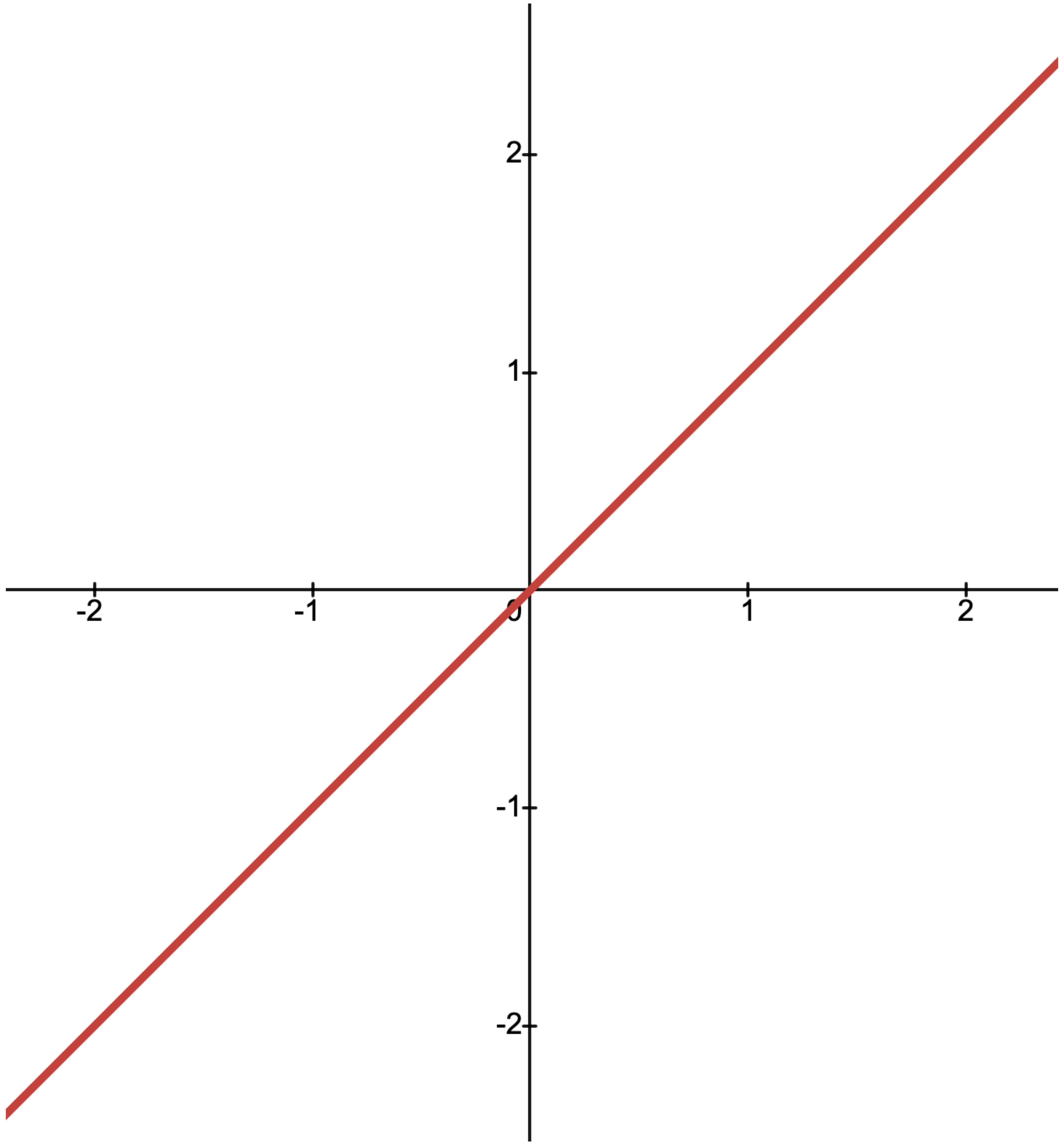
$$y = x$$

Implicit

$$0 = x - y$$

Parametric

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ t \end{pmatrix}$$

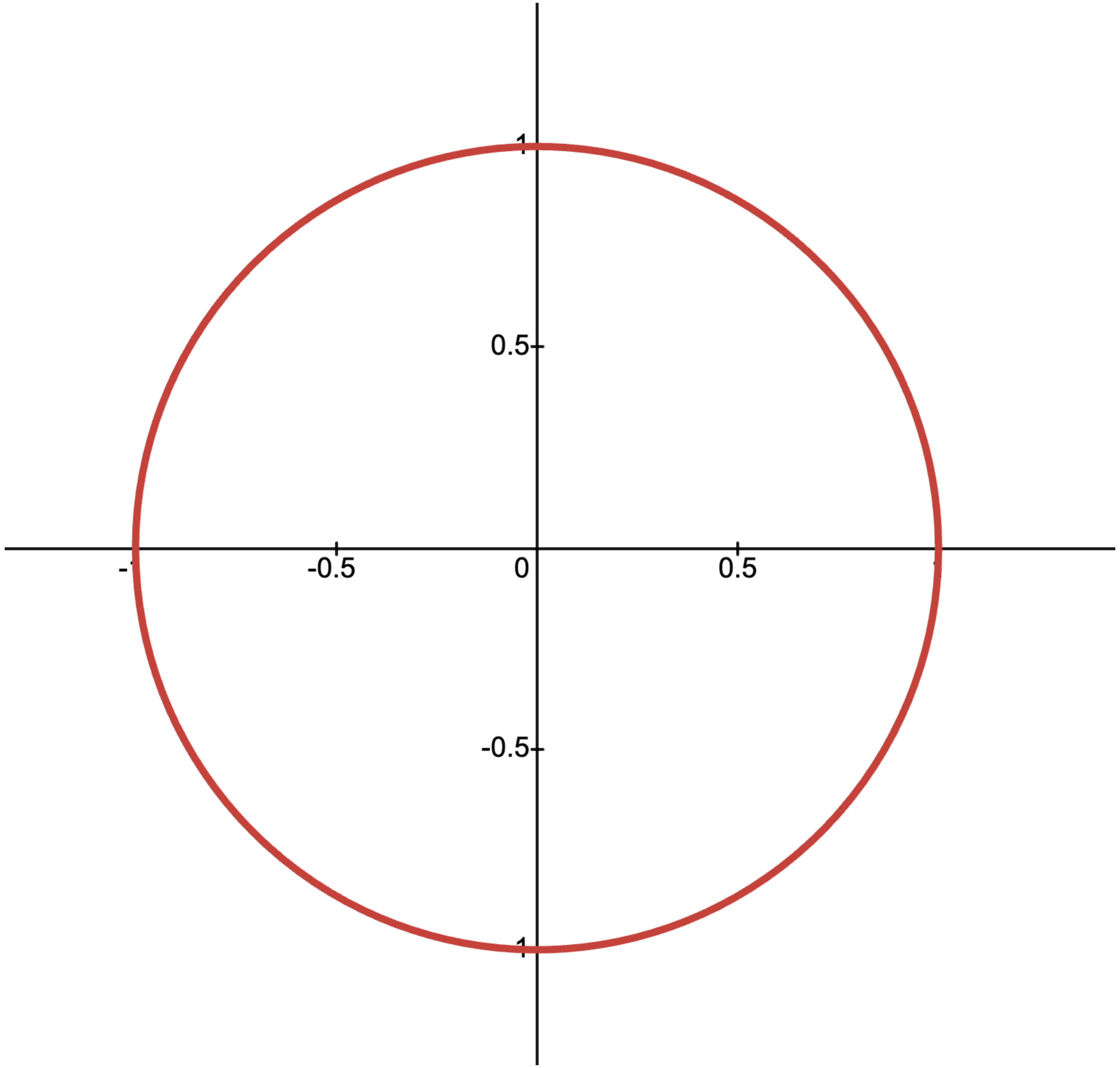


Curve Representations

Explicit

Implicit

Parametric



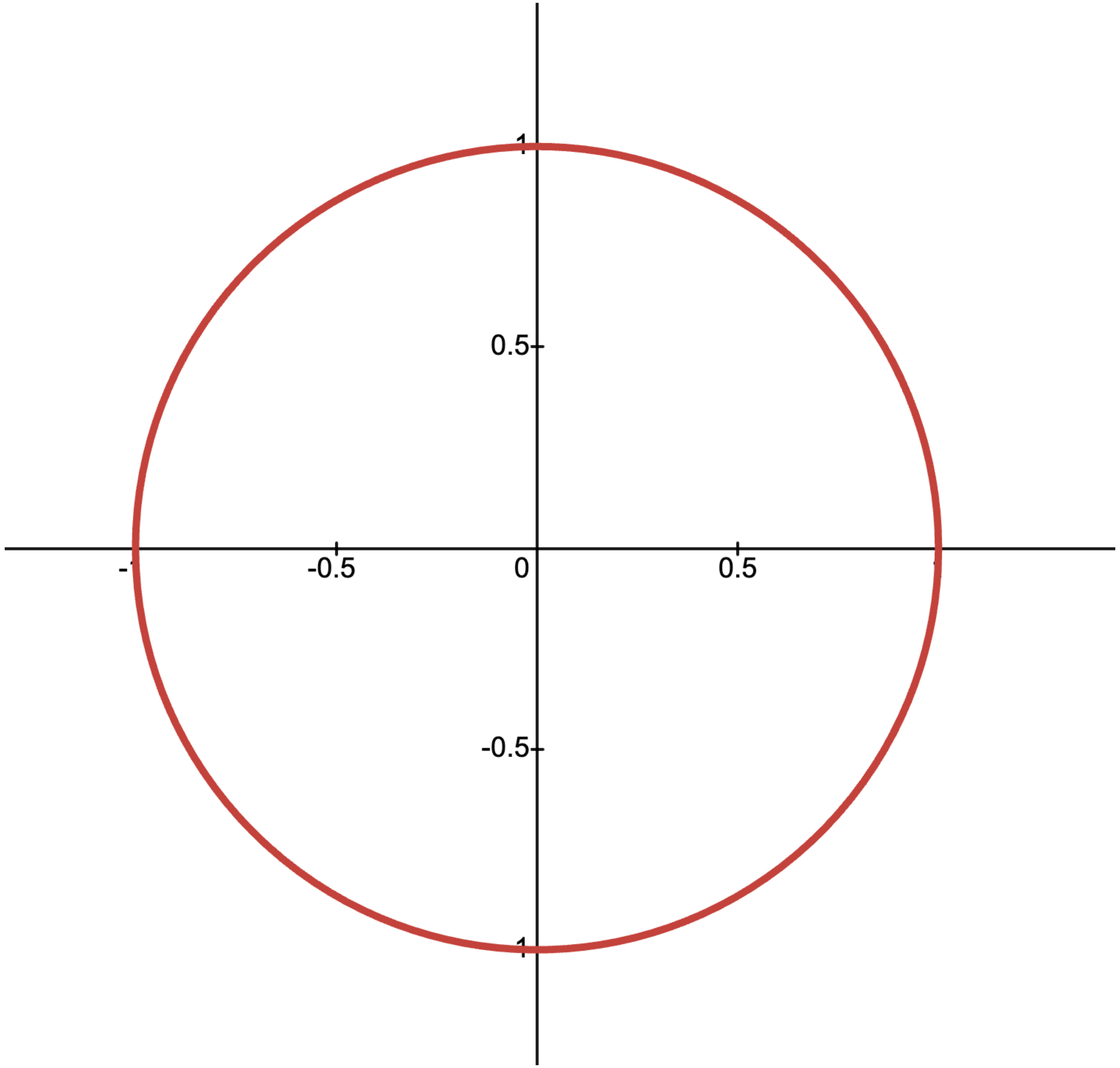
Curve Representations

Explicit

Not possible

Implicit

Parametric



Curve Representations

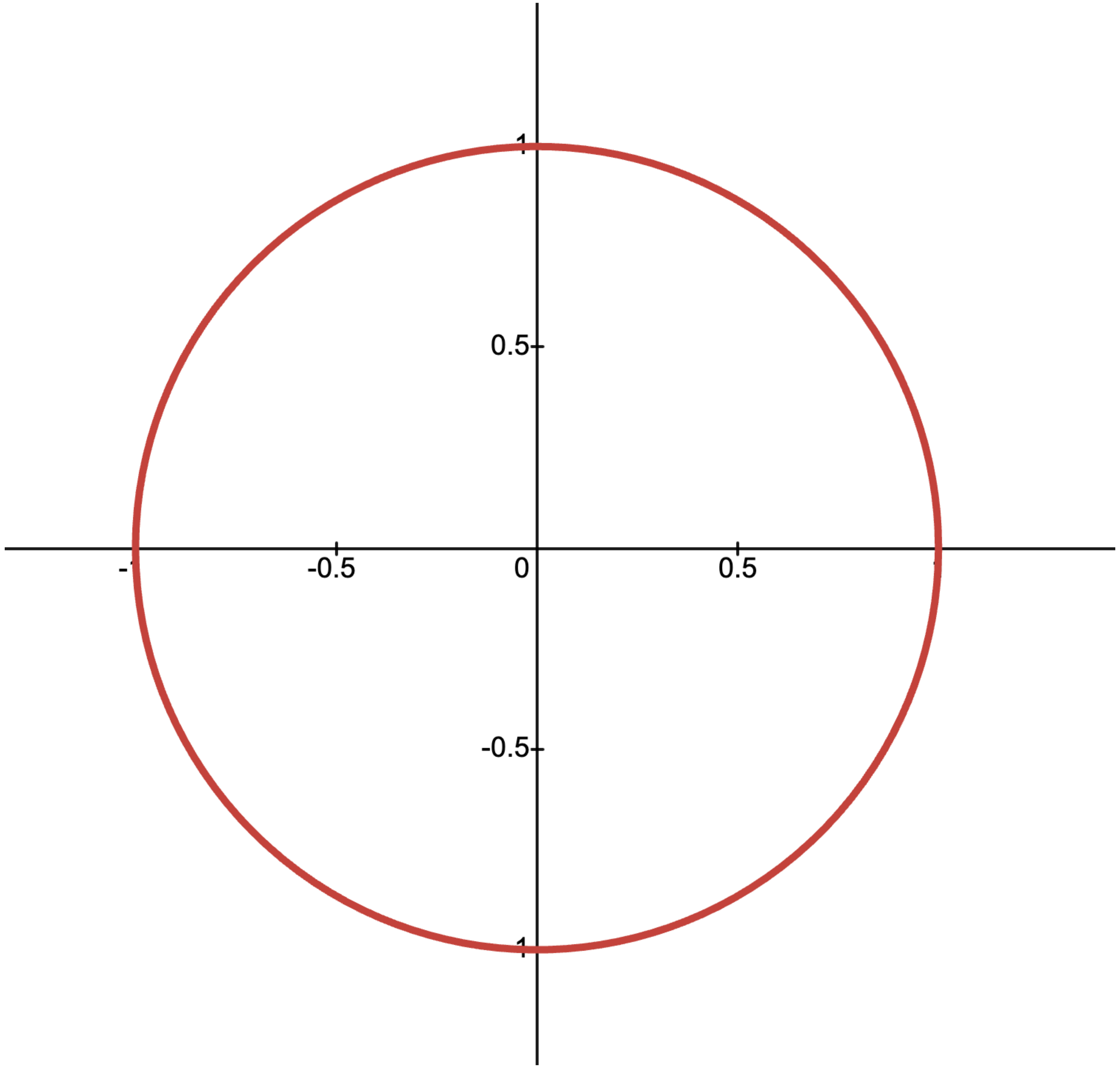
Explicit

Not possible

Implicit

$$0 = x^2 + y^2 - 1$$

Parametric



Curve Representations

Explicit

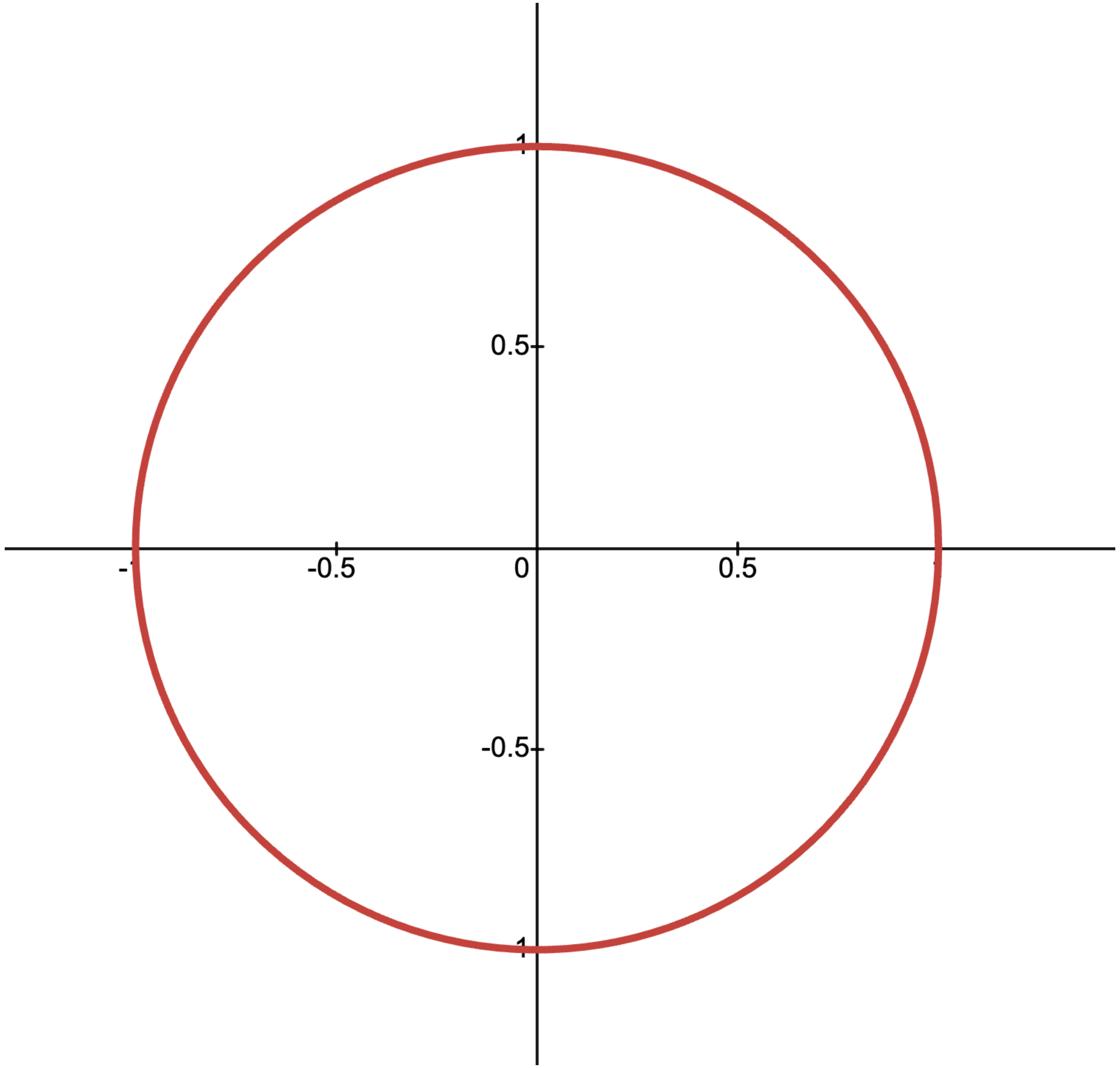
Not possible

Implicit

$$0 = x^2 + y^2 - 1$$

Parametric

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cos(t) \\ sin(t) \end{pmatrix}$$



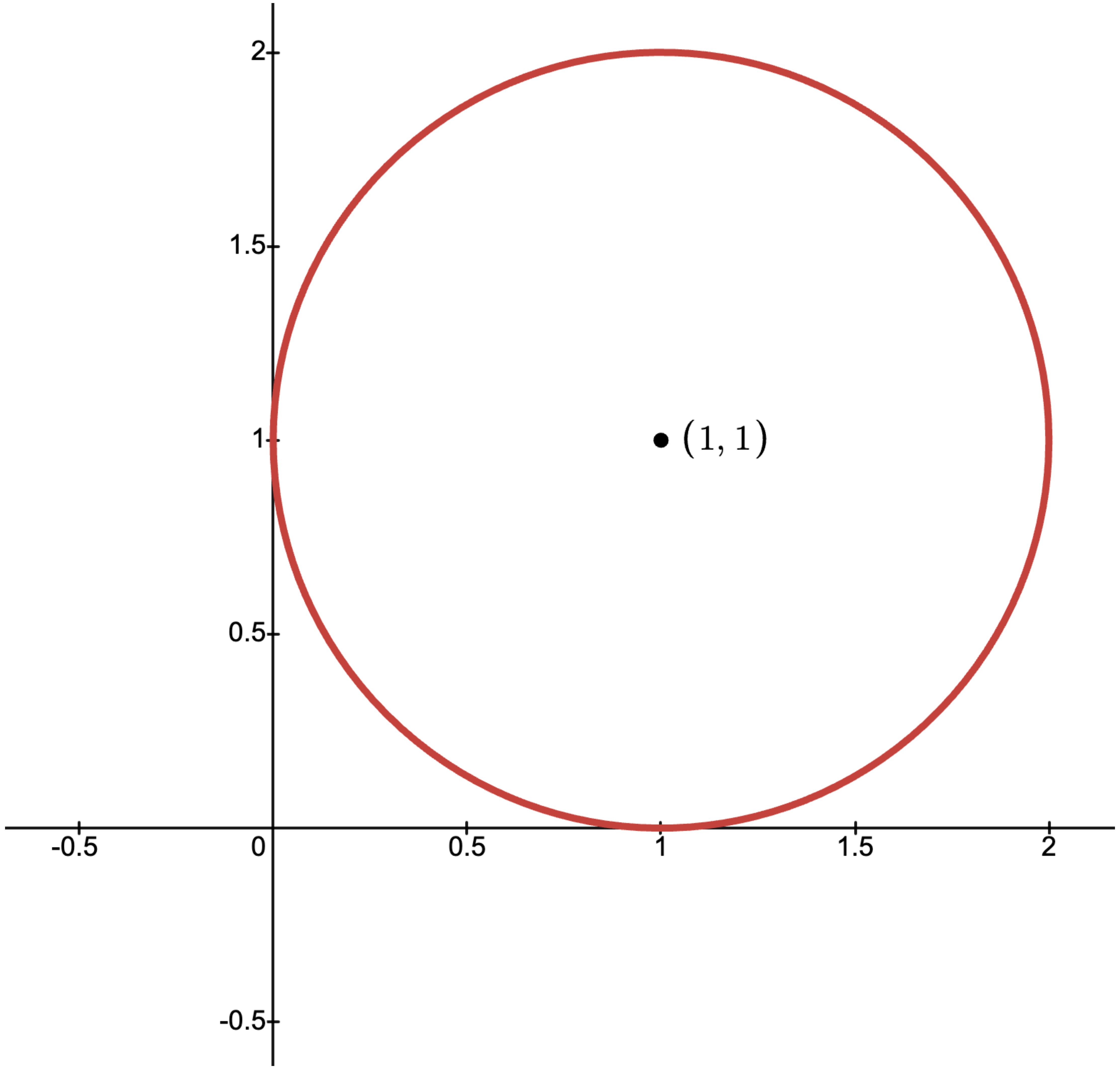
Curve Representations

Explicit

Not possible

Implicit

Parametric



Curve Representations

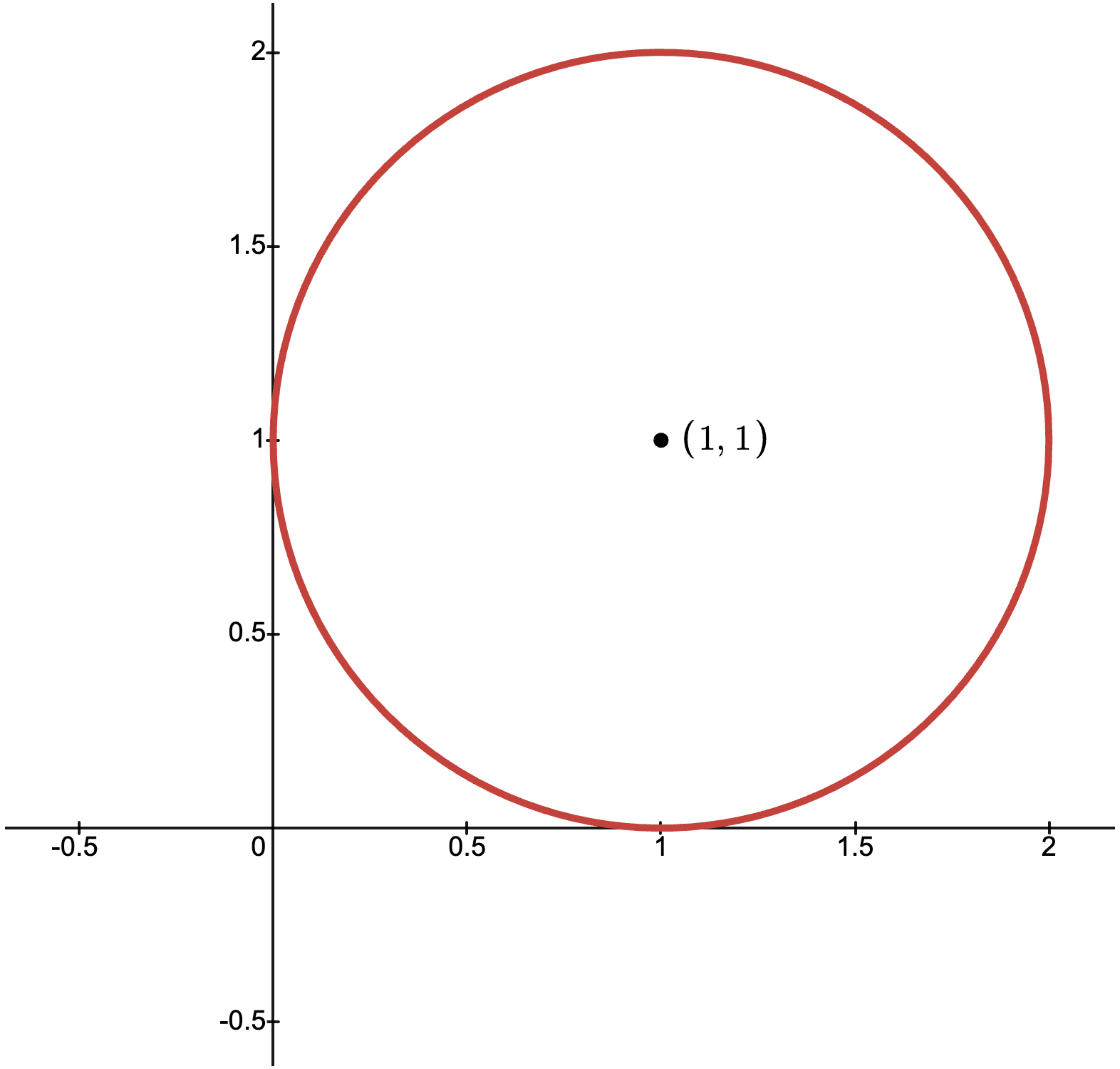
Explicit

Not possible

Implicit

$$0 = (x-1)^2 + (y-1)^2 - 1$$

Parametric



Curve Representations

Explicit

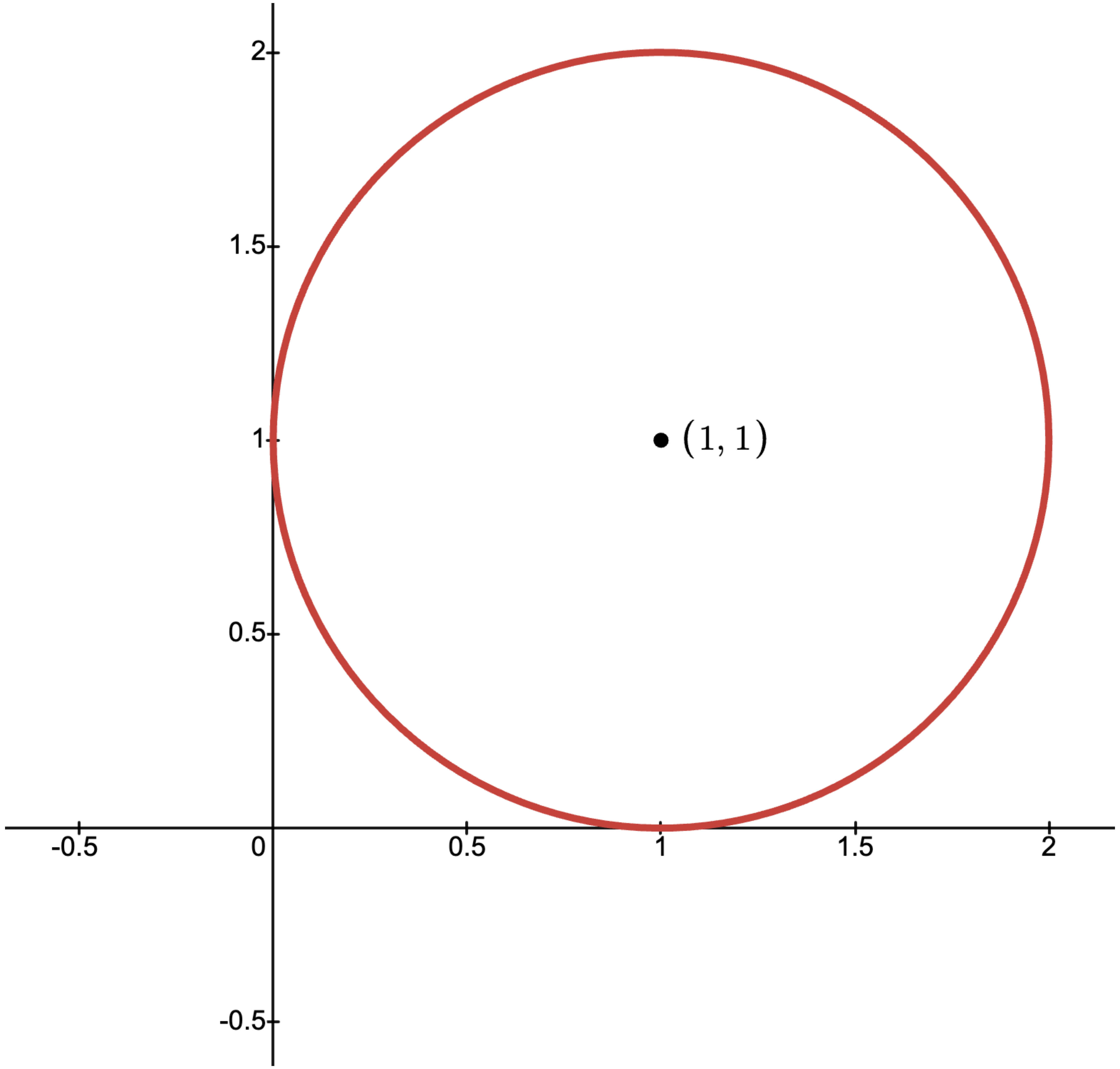
Not possible

Implicit

$$0 = (x-1)^2 + (y-1)^2 - 1$$

Parametric

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cos(t) + 1 \\ sin(t) + 1 \end{pmatrix}$$

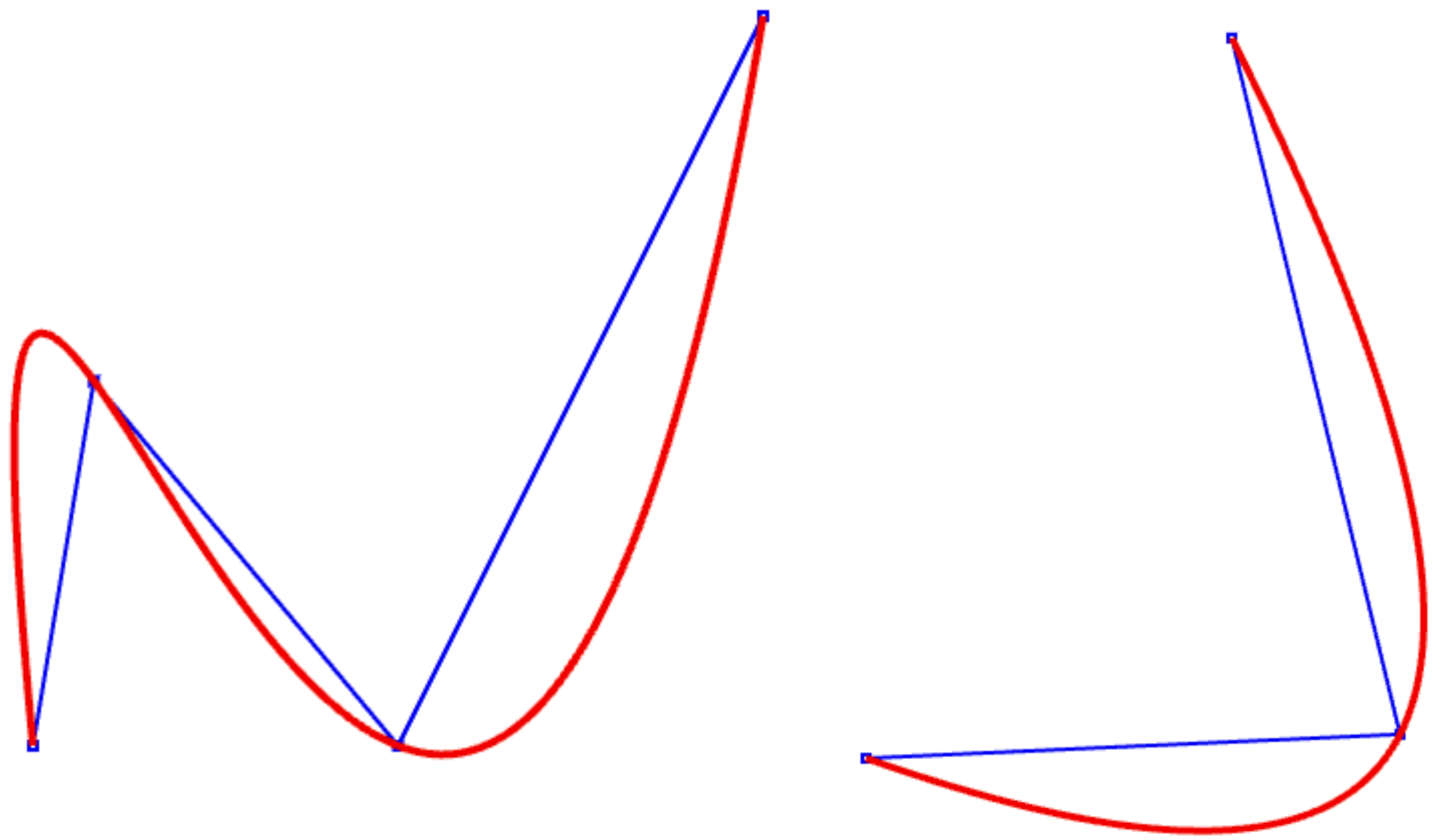


Lagrange Polynomials

Given a set of $k+1$ pairs of point positions $x_i \in \mathbf{R}^N$ and corresponding parameter values $t_i : (t_0, x_0), (t_1, x_1), \ldots (t_k, x_k)$ we can use the following polynomial function $F(t)$ up to degree k to create a curve in $\mathbf{R}^N$ that interpolates them:

$$F(t) = \sum_{i=0}^{k} L_i(t) \cdot x_i$$

$$L_i(t) = \prod_{0 \le m \le k,\, m \ne i} \frac{t - t_m}{t_i - t_m}$$



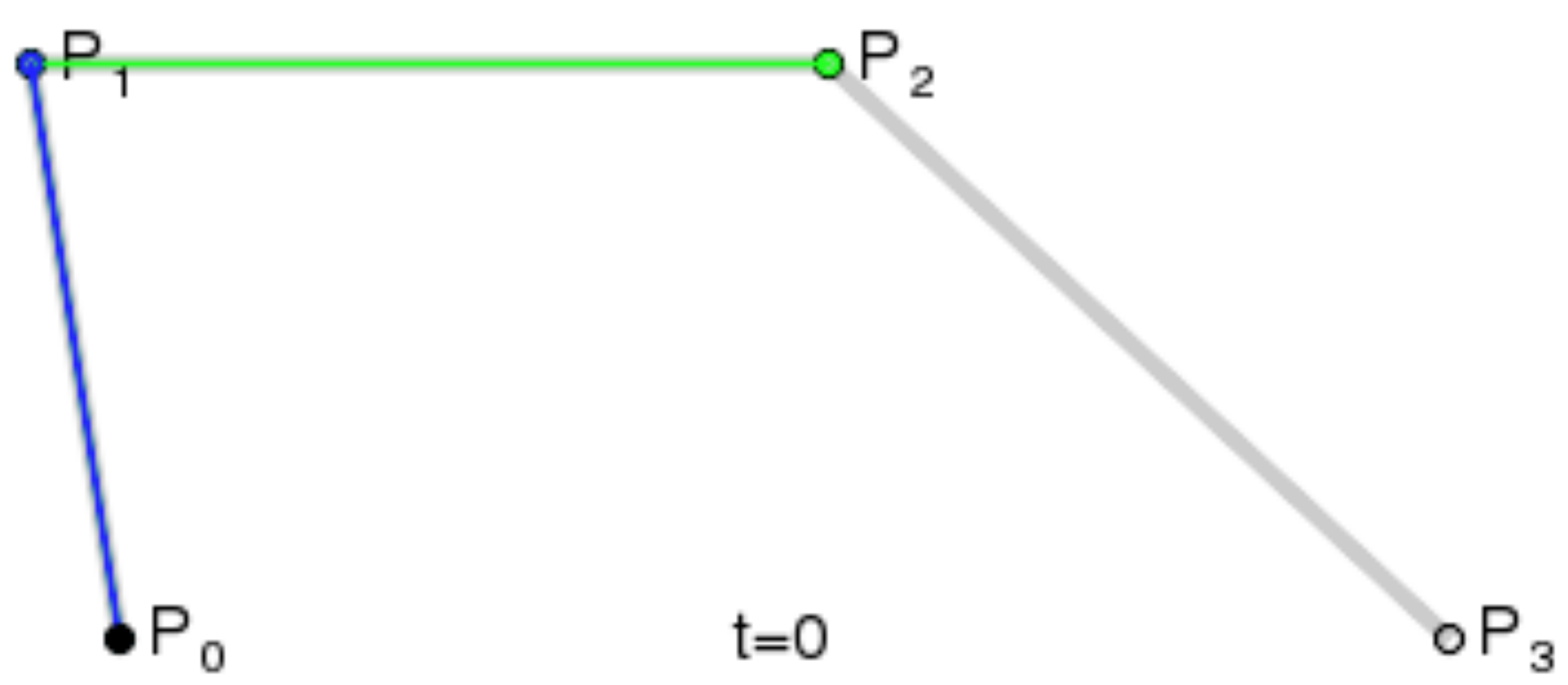
ibiblio

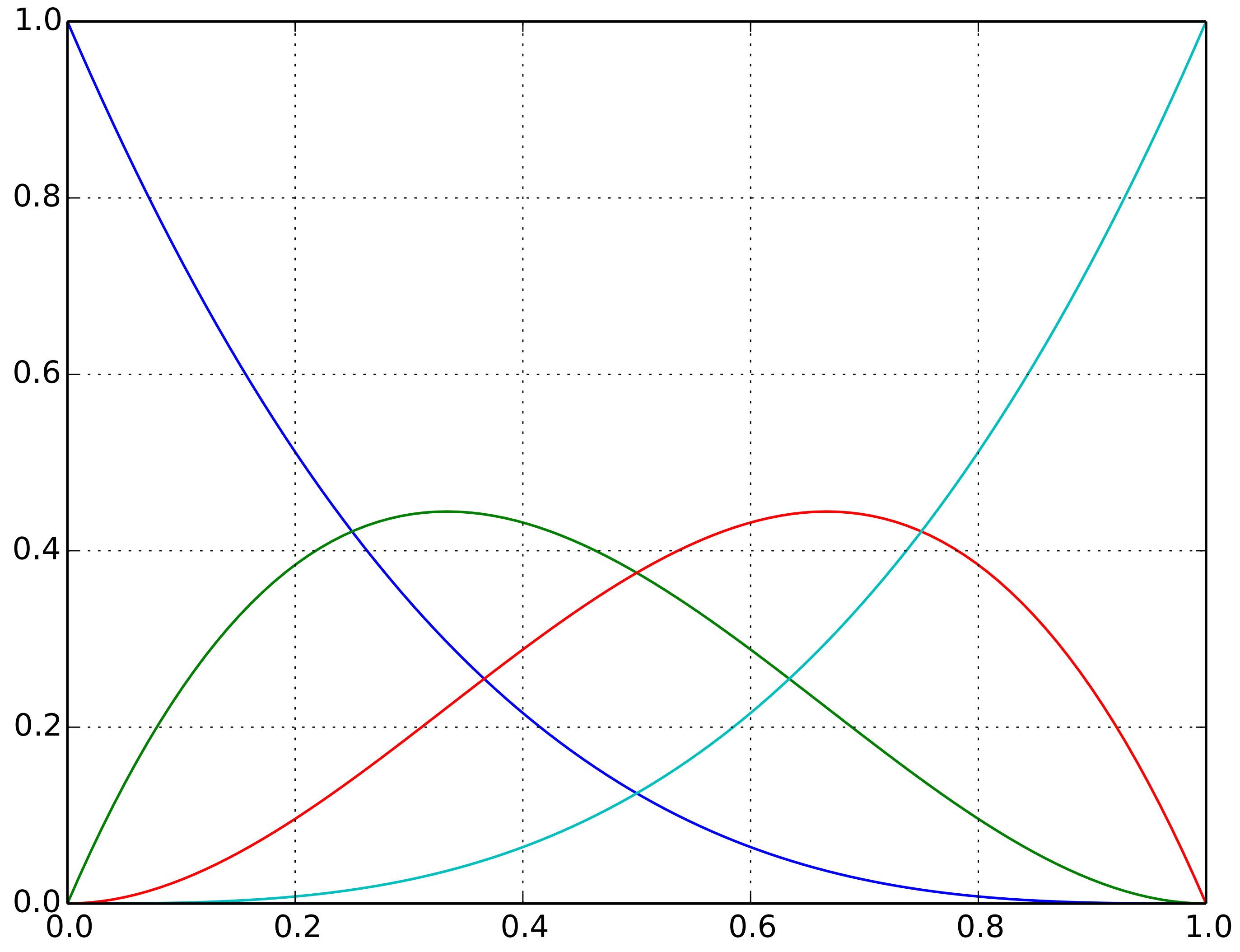
Bézier Basis Functions

Given a set of k pairs of point positions $x_i \in \mathbf{R}^N$, we can use the following polynomial function $F(t)$ up to degree k to create a curve in $\mathbf{R}^N$ that interpolates them:

$$F(t) = \sum_{i=0}^{k} B_i(t) \cdot x_i$$

$$B_i(t) = \binom{k}{i}(1-t)^{k-i} \cdot t^i$$





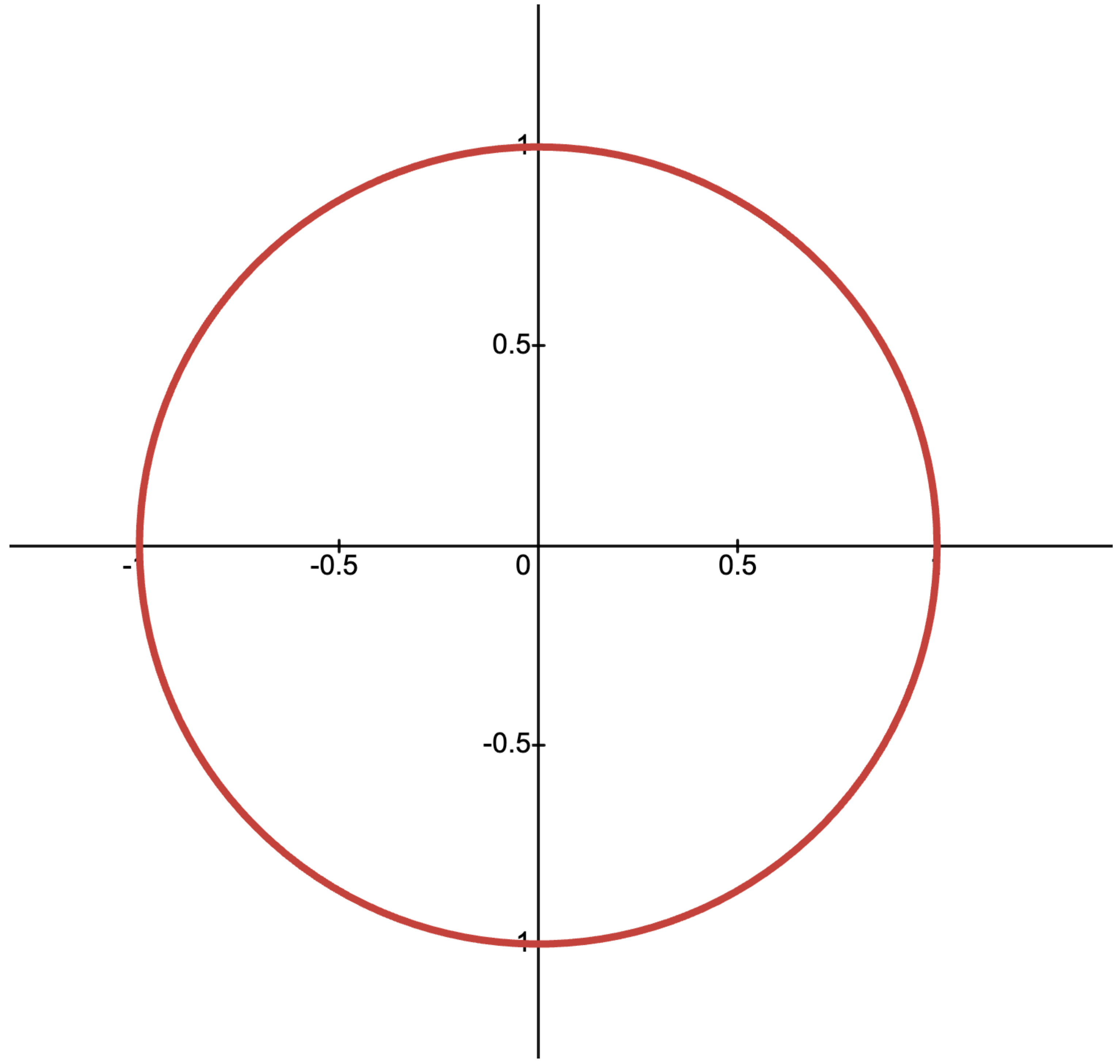
Wikipedia

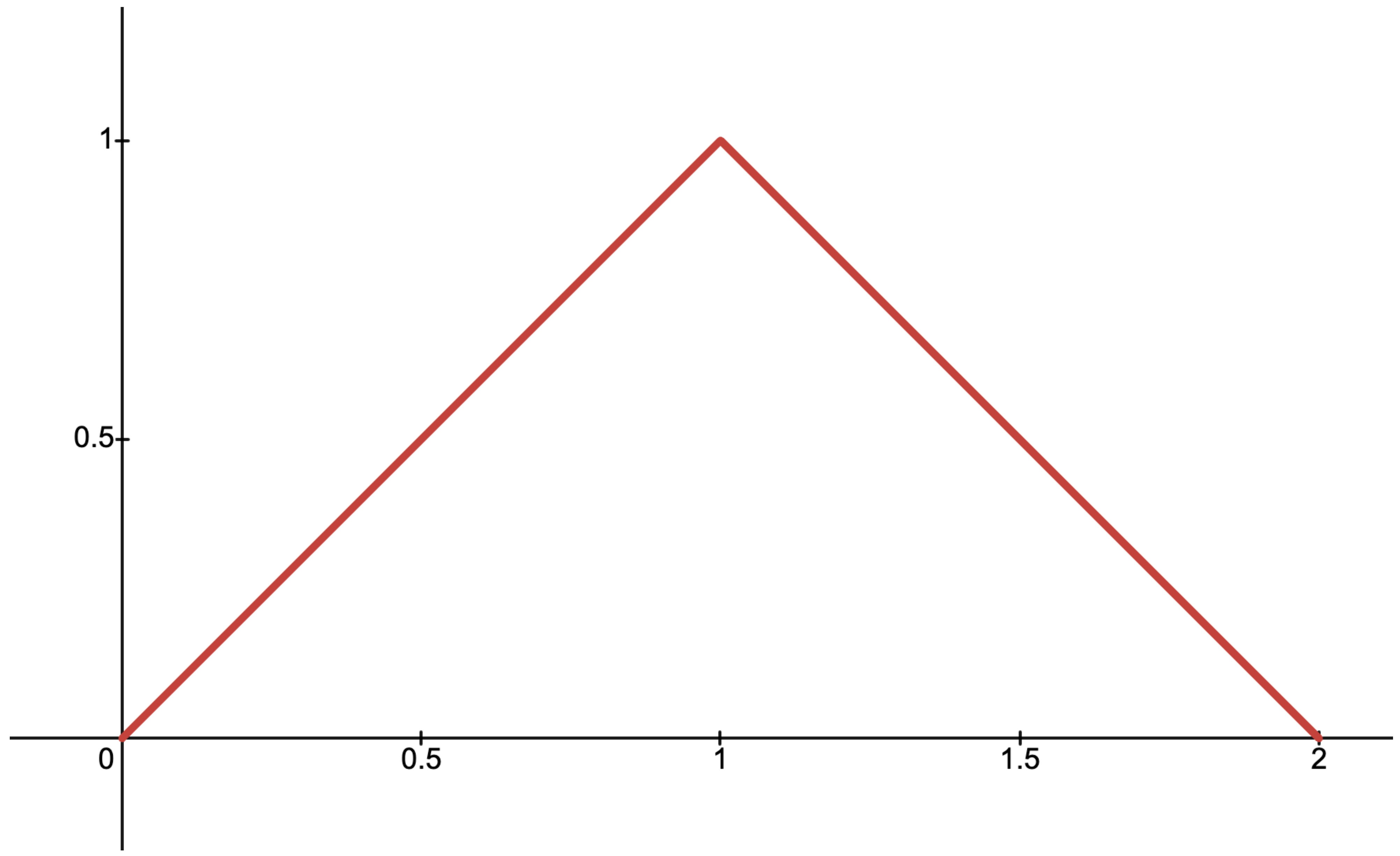
Bézier curves

What kinds of curves ***can't*** be represented using control points interpolated using Bézier Bases?

Bézier curves

What kinds of curves ***can't*** be represented using control points interpolated using Bézier Bases?





Bézier curves

What kinds of curves ***can't*** be represented using control points interpolated using Bézier Bases?

Any functions with discontinuities, any functions that cannot be represented as a finite polynomial (sine/cosine)

Recall that sine has a Taylor expansion - to represent as a polynomial it requires infinite degrees.

Bézier curves

Given the points:

$x_0 = (6,0)$

$x_1 = (4,3)$

$x_2 = (2,3)$

What are the basis functions $B_i(t)$?

What is the resulting curve?

$$F(t) = \sum_{i=0}^{k} B_i(t) \cdot x_i$$

$$B_i(t) = \binom{k}{i}(1-t)^{k-i} \cdot t^i$$

$k = 2$

Bézier Bases

$$
\begin{aligned}
B_0(t) &= \binom{k}{i}(1-t)^{k-i} \cdot t^i \\
&= \binom{2}{0}(1-t)^{2-0} \cdot t^0 \\
&= \frac{2!}{0!(2-0)!}(1-t)^2 \cdot 1 \\
&= \frac{2!}{2!} \cdot (1-t)^2 \\
&= (1-t)^2
\end{aligned}
$$

$k = 2$

Bézier Bases

$$
\begin{aligned}
B_1(t) &= \binom{k}{i}(1-t)^{2-i} \cdot t^i \\
&= \binom{2}{1}(1-t)^{2-1} \cdot t^1 \\
&= \frac{2!}{1!(2-1)!}(1-t)^1 \cdot t \\
&= 2t \cdot (1-t)
\end{aligned}
$$

$$k = 2$$

Bézier Bases

$$\begin{aligned}
B_2(t) &= \binom{k}{i}(1-t)^{k-i} \cdot t^i \\
&= \binom{2}{0}(1-t)^{2-2} \cdot t^2 \\
&= \frac{2!}{2!(2-2)!}(1-t)^0 \cdot t^2 \\
&= \frac{1}{1} \cdot 1 \cdot t^2 \\
&= t^2
\end{aligned}$$

Bézier Bases

$$F(t) = \underbrace{(1-t)^2 \begin{bmatrix}6\\0\end{bmatrix}}_{x_0} + 2t(1-t)\underbrace{\begin{bmatrix}4\\3\end{bmatrix}}_{x_1} + t^2 \underbrace{\begin{bmatrix}2\\3\end{bmatrix}}_{x_2}$$