CPSC 424 - Tutorial 2

September 24, 2026

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Weekly Quiz Questions

Each week, post a question-answer pair to Piazza as a private note to instructors (due the following Monday)

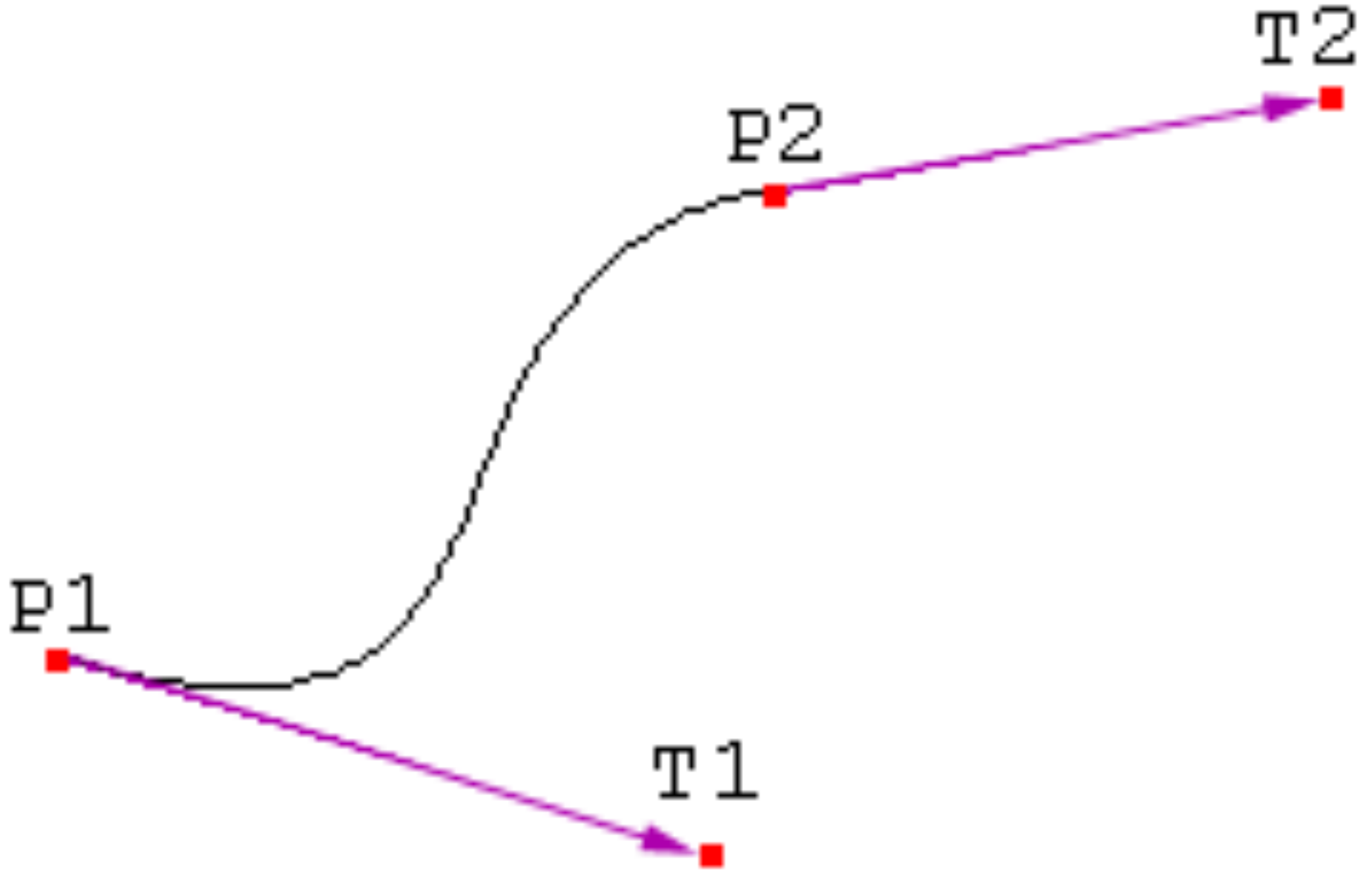
Weekly quiz on Canvas due each Friday (beginning from next week)

Hermite Curves

Curve formed from 2 positions P_0, P_1 and their tangents T_0, T_1

The curve is formulated as
$F(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$

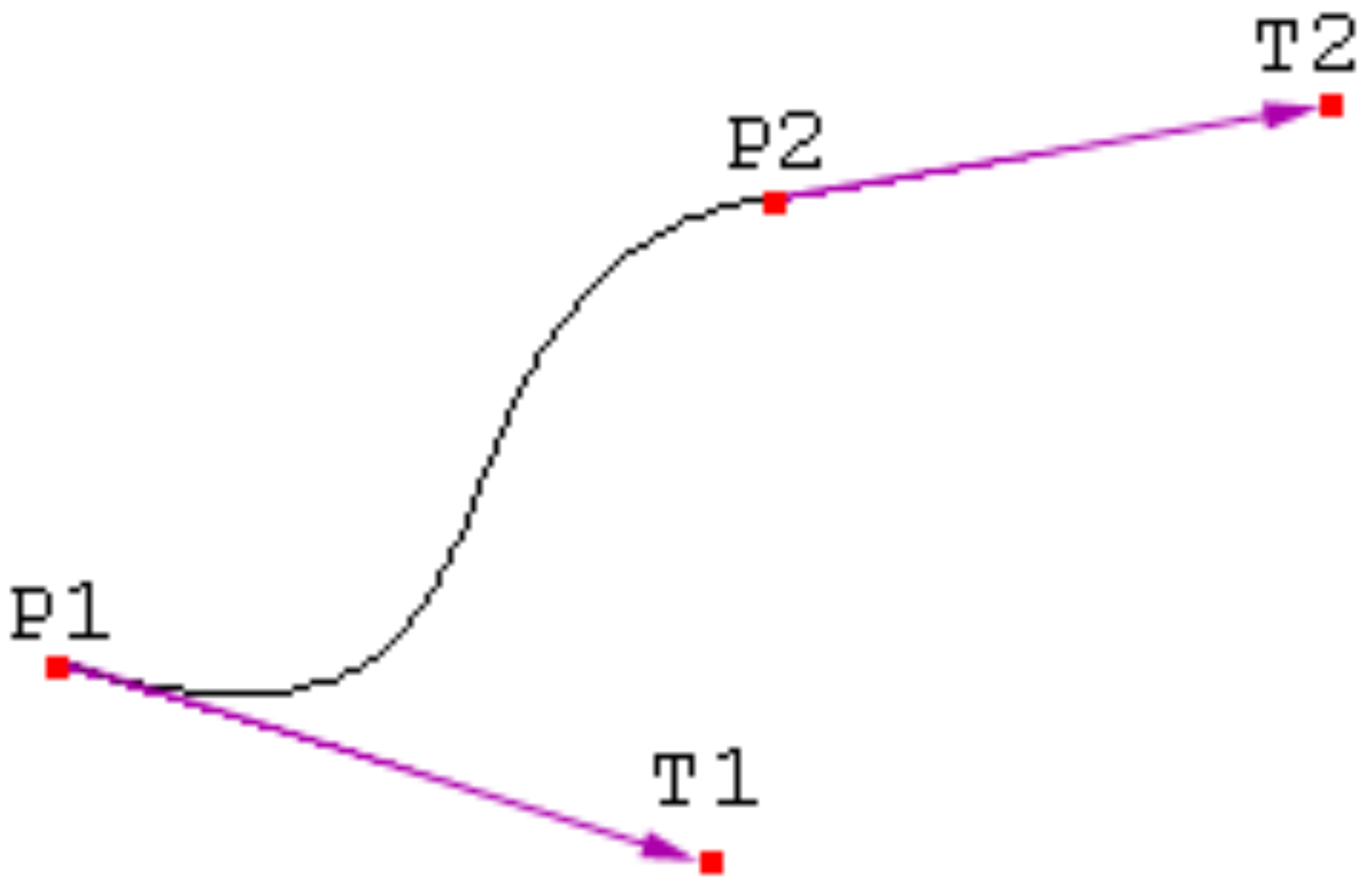
Such that
$F(0) = P_0, F(1) = P_1, F'(0) = T_0, F'(1) = T_1$



https://www.cubic.org/docs/hermite.htm

curve	F(0)	F(1)	F'(0)	F'(1)
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

Hermite Curves



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$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

Hermite Curves

Derivation

Since Hermite curves are cubic polynomials, we represent them as:

$$F(t) = at^3 + bt^2 + ct + d$$

And its derivative as:

$$F'(t) = 3at^2 + 2bt + c$$

Hermite Curves

Derivation

$$F(t) = at^3 + bt^2 + ct + d$$

$$F'(t) = 3at^2 + 2bt + c$$

Recall that for our curves we constraint their position and tangent at $t = 0, t = 1$

Hermite Curves
Derivation

$$F(t) = at^3 + bt^2 + ct + d$$

$$F'(t) = 3at^2 + 2bt + c$$

Recall that for our curves we constraint their position and tangent at $t = 0, t = 1$

$$F(0) = a(0) + b(0) + c(0) + d = d$$

$$F(1) = a + b + c + d$$

$$F'(0) = c$$

$$F'(1) = 3a + 2b + c$$

Hermite Curves

Derivation

$$F(0) = a(0) + b(0) + c(0) + d = d$$

$$F(1) = a + b + c + d$$

$$F'(0) = c$$

$$F'(1) = 3a + 2b + c$$

And as a matrix:

$$\begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Hermite Curves

Derivation

Performing some matrix operations we can set it equal to a, b, c, d

$$\begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

Hermite Curves

Derivation

Performing some matrix operations we can set it equal to a, b, c, d

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

And after taking the inverse of the matrix, we get the Hermite matrix, H_M

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = H_M \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

Hermite Curves

Derivation

Performing some matrix operations we can set it equal to a, b, c, d

And after taking the inverse of the matrix, we get the Hermite matrix, H_M

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = H_M \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

Hermite Curves

Derivation

Recall that F(t) can also be represented as a matrix multiplication:

$$F(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$F(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

Hermite Curves

Derivation

After multiplying out and grouping by P_0, P_1, T_0, T_1 we get our blending polynomials $h(t)$

$$F(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P(0) \\ P(1) \\ T(0) \\ T(1) \end{bmatrix}$$

$$F(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

$$\begin{aligned} h_{00}(t) &= 2t^3 - 3t^2 + 0t + 1 \\ h_{01}(t) &= -2t^3 + 3t^2 + 0t + 0 \\ h_{10}(t) &= t^3 - 2t^2 + t + 0 \\ h_{11}(t) &= t^3 - t^2 + 0t + 0. \end{aligned}$$

Hermite Curves

Derivation

$$F(t) = at^3 + bt^2 + ct + d$$

$$F'(t) = 3at^2 + 2bt + c$$

Recall that for our curves we constraint their position and tangent at $t = 0, t = 1$

$$\begin{aligned}
h_{00}(t) &= 2t^3 - 3t^2 + 0t + 1 \\
h_{01}(t) &= -2t^3 + 3t^2 + 0t + 0 \\
h_{10}(t) &= t^3 - 2t^2 + t + 0 \\
h_{11}(t) &= t^3 - t^2 + 0t + 0.
\end{aligned}$$

Hermite Curves
Derivation

$$F(t) = at^3 + bt^2 + ct + d$$

$$F'(t) = 3at^2 + 2bt + c$$

Recall that for our curves we constraint their position and tangent at $t = 0, t = 1$

$$H(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

$$h_{00}(t) = t^2(2t-3)+1 \qquad h_{01}(t) = -t^2(2t-3)$$

$$h_{10}(t) = t(t-1)^2 \qquad h_{11}(t) = t^2(t-1)$$

Hermite vs. Bézier Curves

Quadratic:

Fundamentally, both describe a quadratic equation ($at^2 + bt + c$)

Same curve can be expressed as a Hermite or Bézier (different controls)

$$H(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

Hermite

$$h_{00}(t) = t^2(2t-3)+1 \qquad h_{01}(t) = -t^2(2t-3)$$

$$h_{10}(t) = t(t-1)^2 \qquad h_{11}(t) = t^2(t-1)$$

Bézier

$$F(t) = (1-t)^2 P_0 + 2(1-t)tP_1 + t^2 P_2$$

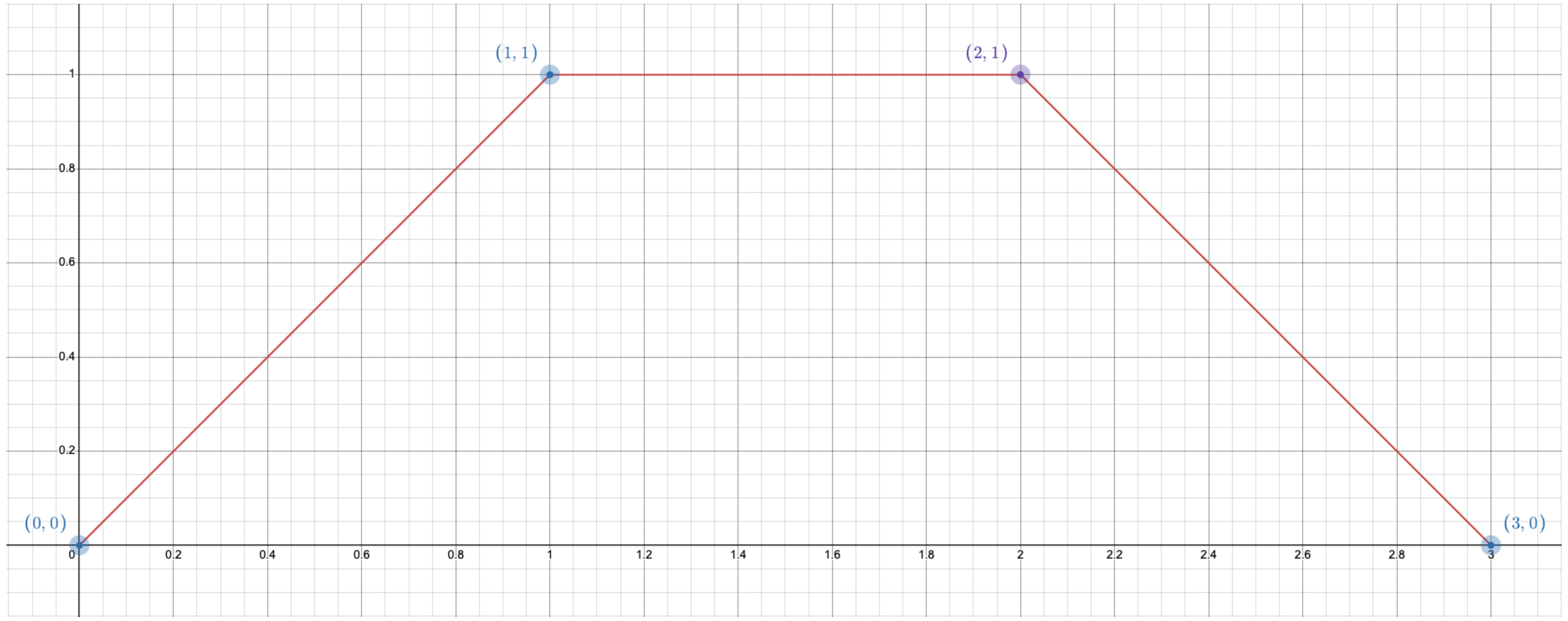
De Casteljau's Algorithm

How can we find points on a Bézier curve without computing bases?

Recursively evaluate by "connecting" t-values on edges

Ex. Find $t = 0.5$ given cubic Bezier curve with control points

$$a = (0,0), \quad b = (1,1), \quad c = (2,1), \quad d = (3,0)$$



$a = (0,0), \quad b = (1,1), \quad c = (2,1), \quad d = (3,0)$

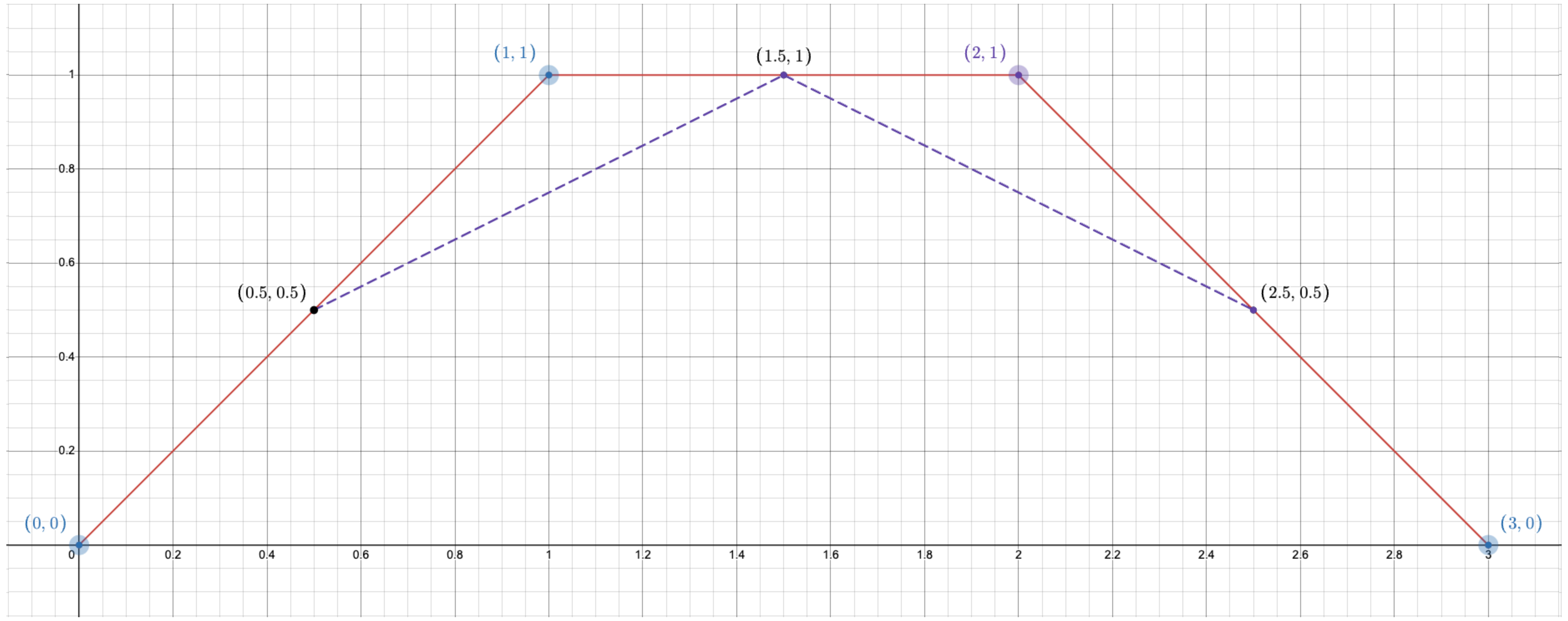
De Casteljau's Algorithm

1. Compute $t = 0.5$ for each of the edges $[a, b], \quad [b, c], \quad [c, d]$

$$e = a + 0.5(b - a) = (0.5,0.5)$$

$$f = b + 0.5(c - b) = (1.5,1)$$

$$g = c + 0.5(d - c) = (2.5,0.5)$$



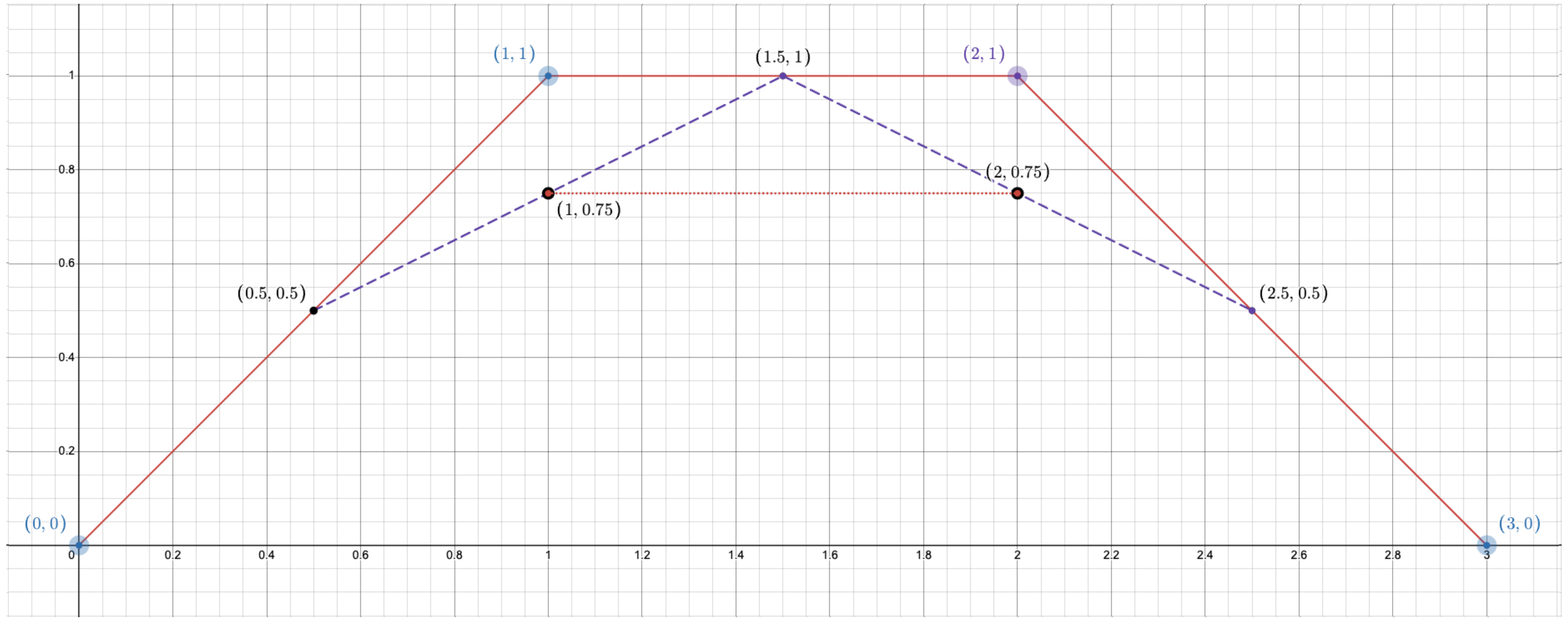
$$a = (0,0), \quad b = (1,1), \quad c = (2,1), \quad d = (3,0)$$
$$e = (0.5,0.5), \quad f = (1.5,1), \quad g = (2.5,0.5)$$

De Casteljau's Algorithm

2. Compute $t = 0.5$ for each of the edges $[e, f], \quad [f, g]$

$$h = e + 0.5(f - e) = (1,0.75)$$

$$i = f + 0.5(g - f) = (2,0.75)$$

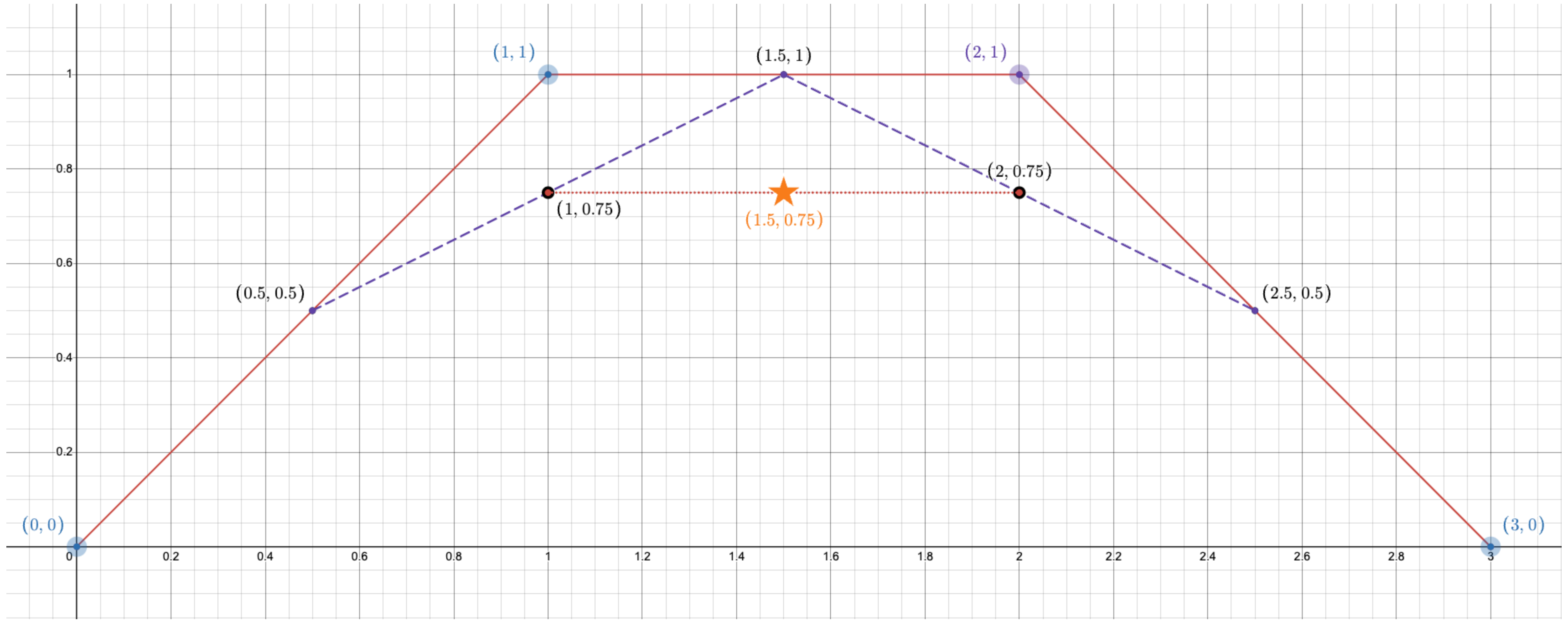


$$a = (0,0), \quad b = (1,1), \quad c = (2,1), \quad d = (3,0)$$
$$e = (0.5,0.5), \quad f = (1.5,1), \quad g = (2.5,0.5)$$
$$h = (1,0.75), \quad i = (2,0.75)$$

De Casteljau's Algorithm

3. Compute $t = 0.5$ for edge $[h, i]$

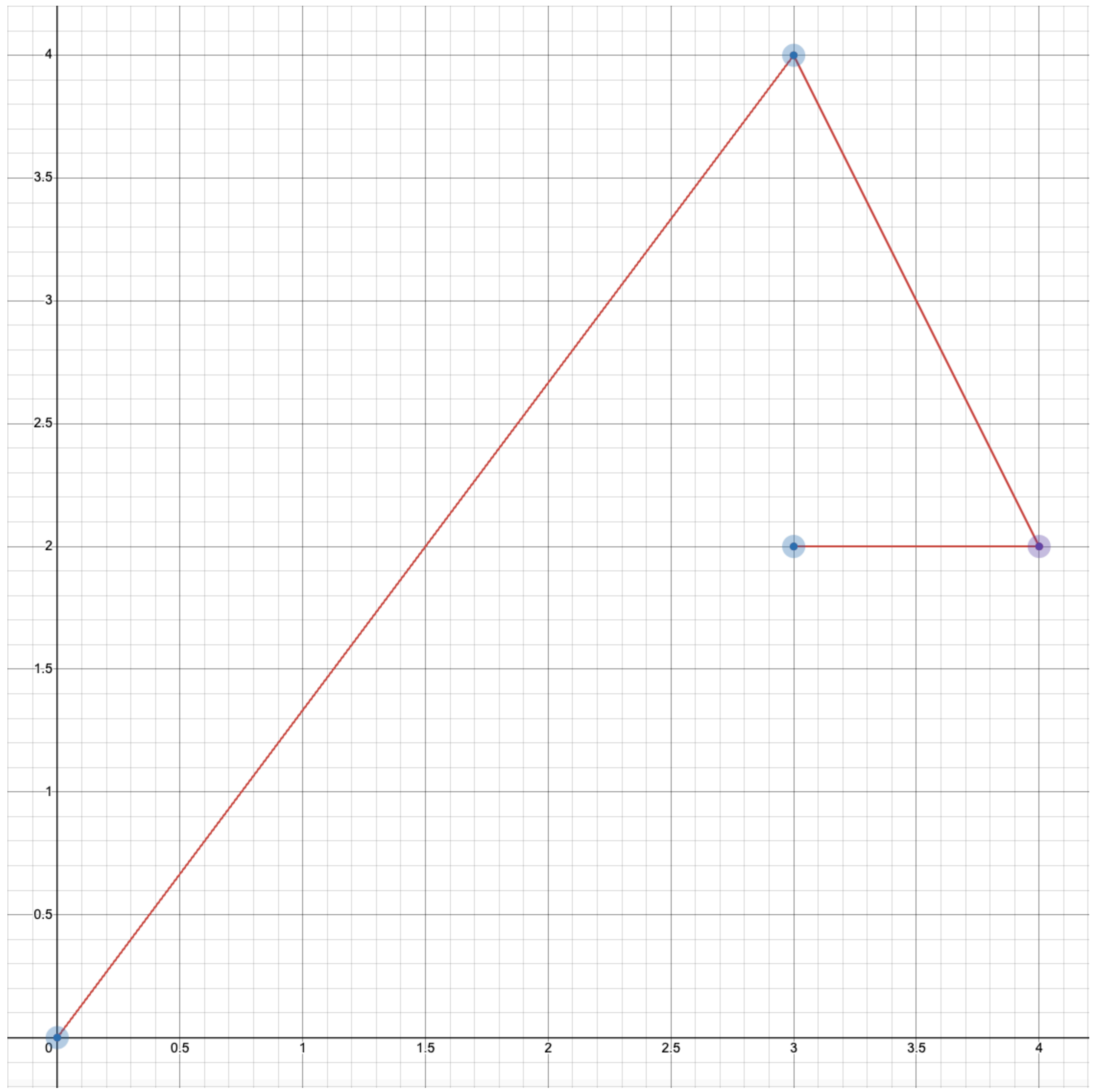
$$j = h + 0.5(i - h) = (1.5,0.75)$$



De Casteljau's Algorithm

Exercise 1.
For the cubic Bézier curve defined by the following 4 control points, find the point at t = 0.25 (use a calculator if needed) :

$$a = (0,0),\ b = (3,4),\ c = (4,2),\ d = (3,2)$$



De Casteljau's Algorithm

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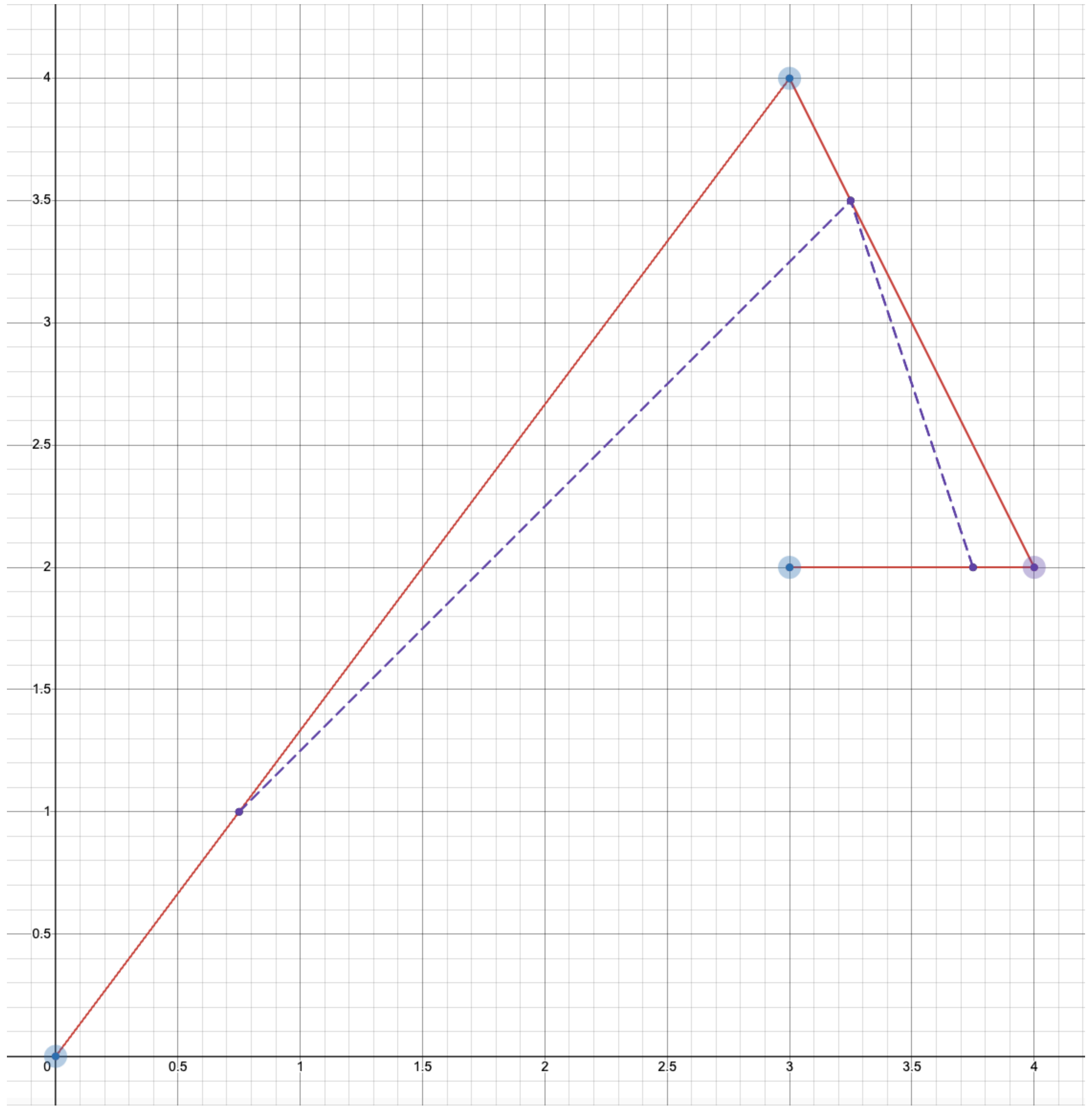
$$a = (0,0),\ b = (3,4),\ c = (4,2),\ d = (3,2)$$

$$e = a + 0.25(b - a) = (0.75,1)$$

$$f = b + 0.25(c - b) = (3.25,3.5)$$

$$g = c + 0.25(d - c) = (3.75,2)$$



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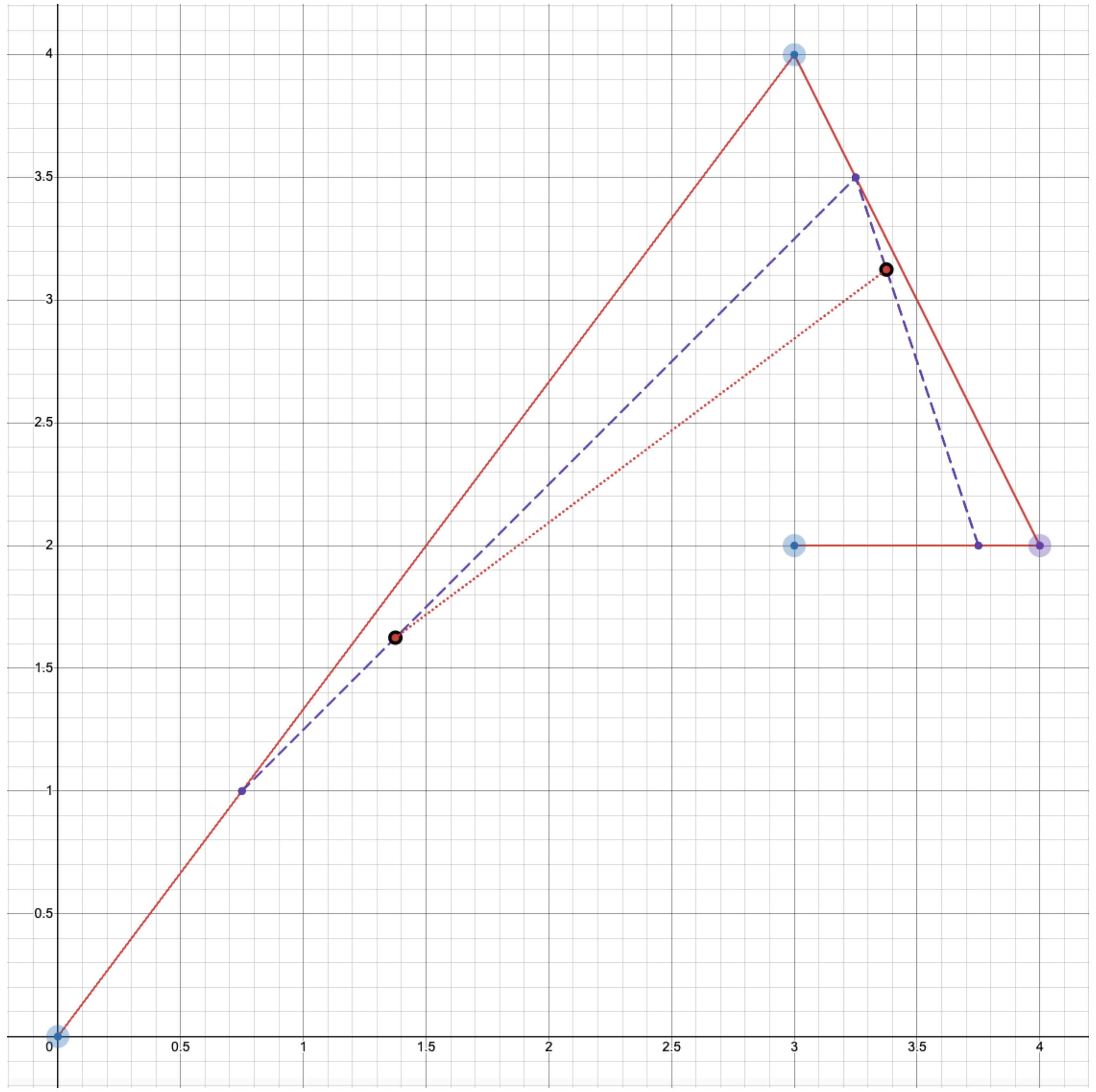
$$a = (0,0), b = (3,4), c = (4,2), d = (3,2)$$

$$e = (0.75,1), f = (3.25,3.5), g = (3.75,2)$$

$$h = e + 0.25(f - e) = (1.375,1.625)$$

$$i = f + 0.25(g - f) = (3.375,3.125)$$



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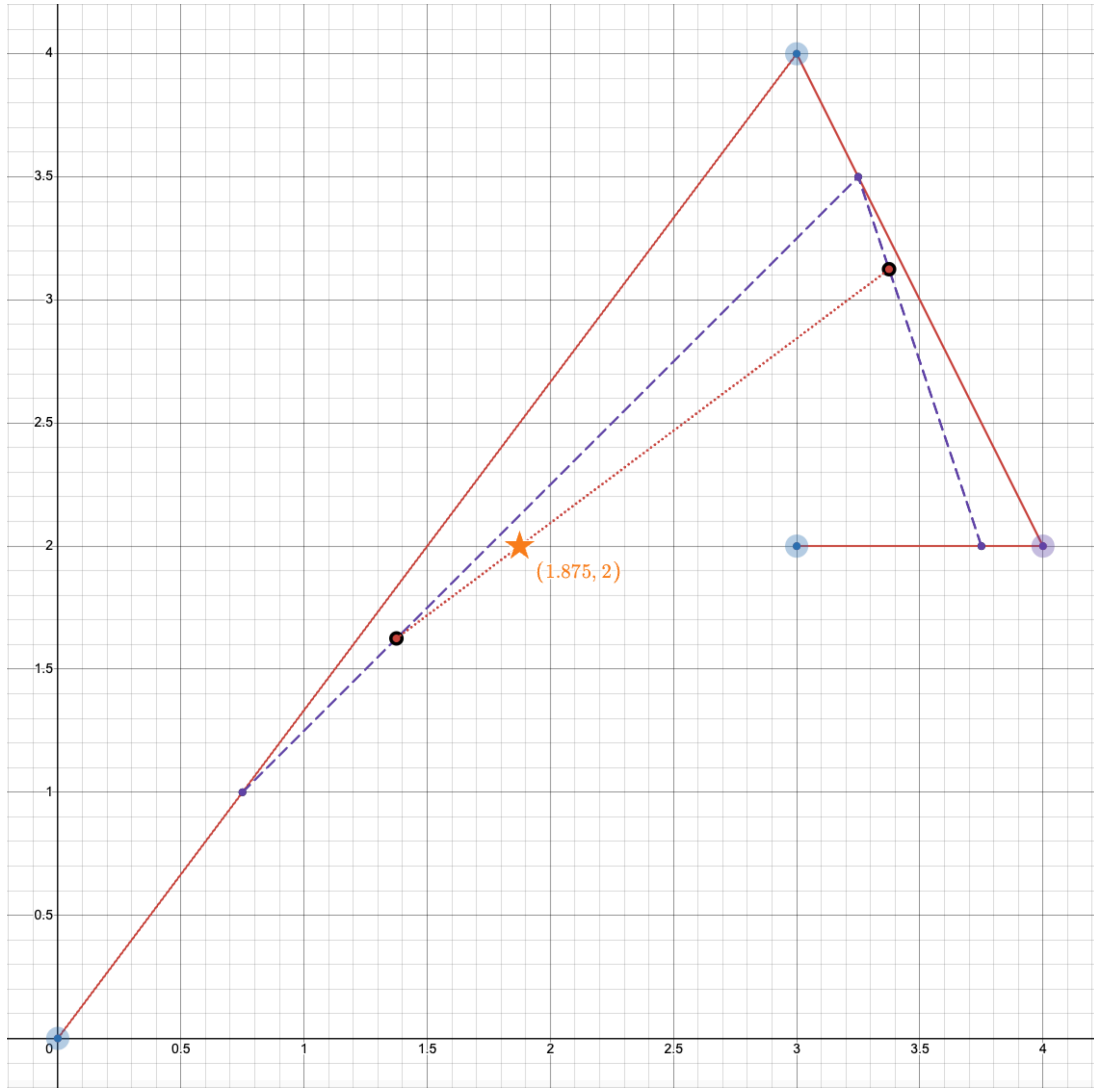
$$e = (0.75,1), f = (3.25,3.5),\ g = (3.75,2)$$

$$h = (1.375,1.625),\ i = (3.375,3.125)$$

$$j = h + 0.25(i - h) = (1.875,2)$$



De Casteljau's Algorithm

Exercise 1.
For the cubic Bézier curve defined by the following 4 control points, find the point at t=0.5 :

$$a = (0,0),\, b = (3,4),\, c = (4,2),\, d = (3,2)$$