

CPSC 424 - Tutorial 3

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TAs: Daniel Cui (dwcui@cs.ubc.ca), Oliver Oxford (oliox@cs.ubc.ca)

Continuity

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Note that if a function is C_k continuous, then it is also C_j continuous for $j < k$.

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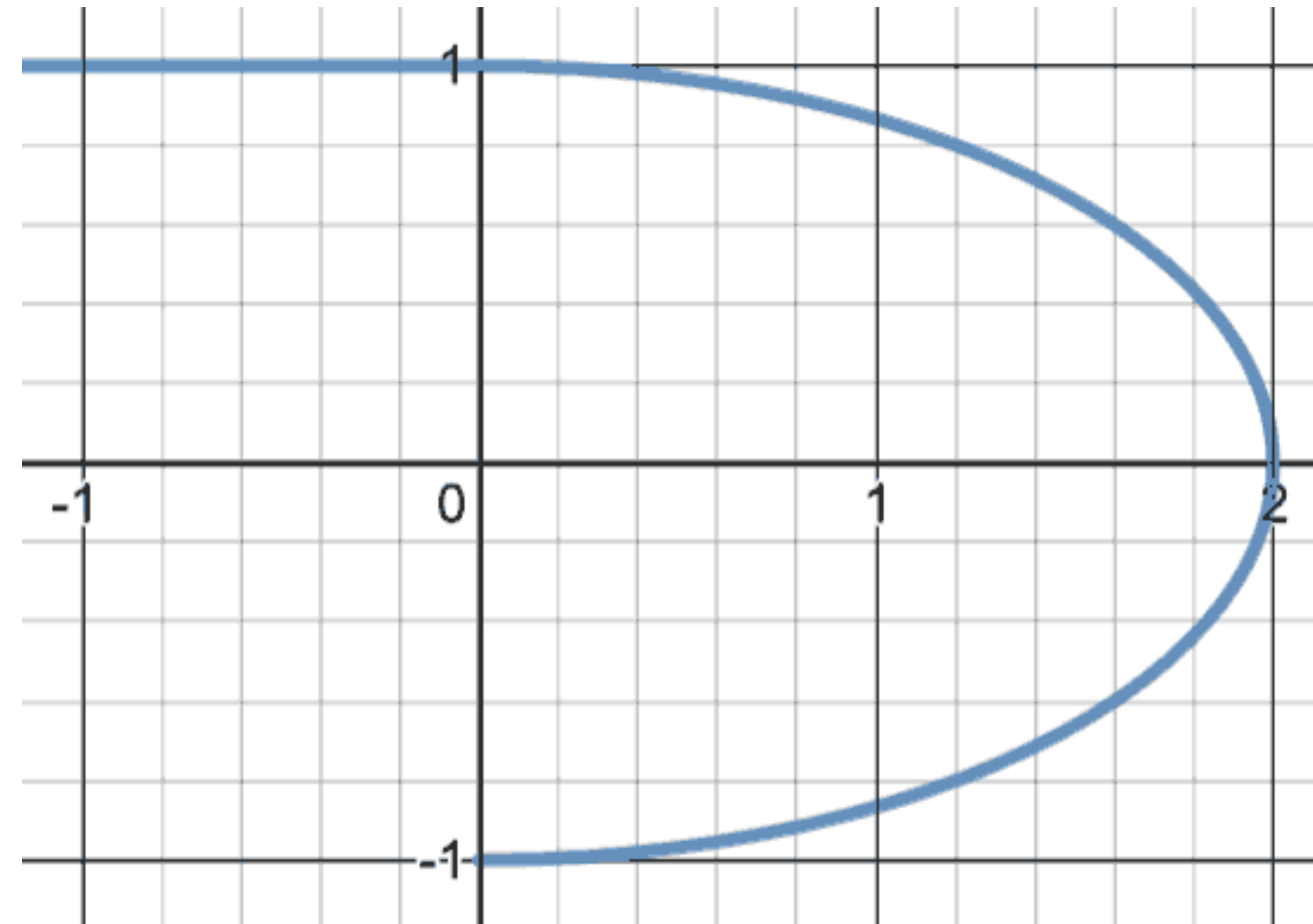
Note that C_k implies G_k continuity.

Continuity

For example, this function here:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{cases} \begin{bmatrix} t \\ 1 \end{bmatrix} & t < 0 \\ \begin{bmatrix} 2 \sin t \\ \cos t \end{bmatrix} & 0 \leq t \leq 1 \end{cases}$$

Is G_1 but not C_1 continuous because its derivative at $t = 0$ from the left is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ but its derivative from the right is $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$



Continuity

Exercise 1.

Determine the parametric continuity (C_1 , C_2 , etc.) and geometric continuity (G_1 , G_2) of each of the curves:

$$f_1(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{cases} \begin{bmatrix} (t+1)^3/3 \\ t^3 \end{bmatrix} & 0 \leq t < 1 \\ \begin{bmatrix} t^2 + 2t - 1/3 \\ t^3/2 + 3t/2 - 1 \end{bmatrix} & 1 \leq t \leq 2 \end{cases}$$

$$f_2(t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{cases} \begin{bmatrix} t^3/6 + t^2/2 + t/2 - 1/6 \\ t^2 \end{bmatrix} & 0 \leq t < 1 \\ \begin{bmatrix} t^2 \\ t^2 \end{bmatrix} & 1 \leq t < 2 \\ \begin{bmatrix} 3t^2/2 + t - 4 \\ t^2 \end{bmatrix} & 2 \leq t \end{cases}$$

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$$\lim_{t \rightarrow 1^-} f_1(t) = \begin{bmatrix} \frac{(1+1)^3}{3} \\ 1^3 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} \\ 1 \end{bmatrix},$$

$$\lim_{t \rightarrow 1^+} f_1(t) = \begin{bmatrix} 1^2 + 2(1) - \frac{1}{3} \\ \frac{1^3}{2} + \frac{3(1)}{2} - 1 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} \\ 1 \end{bmatrix} \implies C^0, G^0.$$

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$$f_1'(t) = \begin{cases} \begin{bmatrix} (t+1)^2 \\ 3t^2 \end{bmatrix} & 0 \leq t < 1, \\ \begin{bmatrix} 2t + 2 \\ \frac{3t^2}{2} + \frac{3}{2} \end{bmatrix} & 1 \leq t \leq 2. \end{cases}$$

$$\lim_{t \rightarrow 1^-} f_1'(t) = \begin{bmatrix} (1+1)^2 \\ 3(1)^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix},$$

$$\lim_{t \rightarrow 1^+} f_1'(t) = \begin{bmatrix} 2(1) + 2 \\ \frac{3(1)^2}{2} + \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \implies C^1, G^1.$$

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$$f_1''(t) = \begin{cases} \begin{bmatrix} 2(t+1) \\ 6t \end{bmatrix} & 0 \leq t < 1, \\ [10pt] \begin{bmatrix} 2 \\ 3t \end{bmatrix} & 1 \leq t \leq 2. \end{cases}$$

$$\lim_{t \rightarrow 1^-} f_1''(t) = \begin{bmatrix} 2(1+1) \\ 6(1) \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix},$$

$$\lim_{t \rightarrow 1^+} f_1''(t) = \begin{bmatrix} 2 \\ 3(1) \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \implies \not C^2, \text{ but } G^2.$$