CPSC 424 - Tutorial 8

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Credits to our awesome former TA Jerry Yin for some of the content of the slides!

Barycentric coordinates

Barycentric coordinates allow us to express the position of any point located on a triangle as a weighted sum of its vertices.

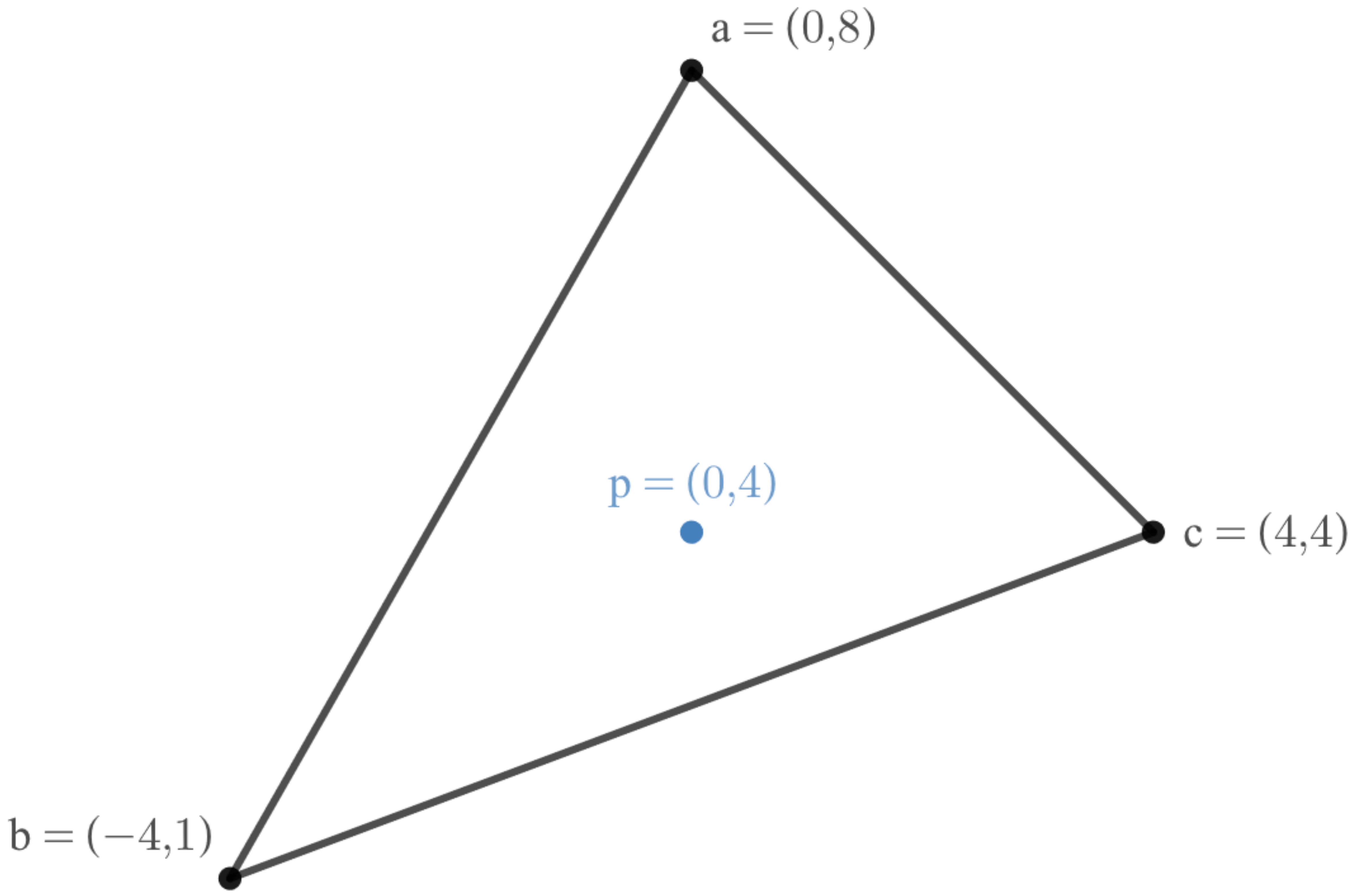
To compute the position of this point P using barycentric coordinates for a triangle ABC, we use the following equation:

$P = \alpha A + \beta B + \gamma C,$ where $\alpha + \beta + \gamma = 1$

Barycentric coordinates

$P = \alpha A + \beta A + \gamma C$, where $\alpha + \beta + \gamma = 1$

But how do we choose the weights α, β, γ?



Barycentric coordinates

$P = \alpha A + \beta A + \gamma C$, where $\alpha + \beta + \gamma = 1$

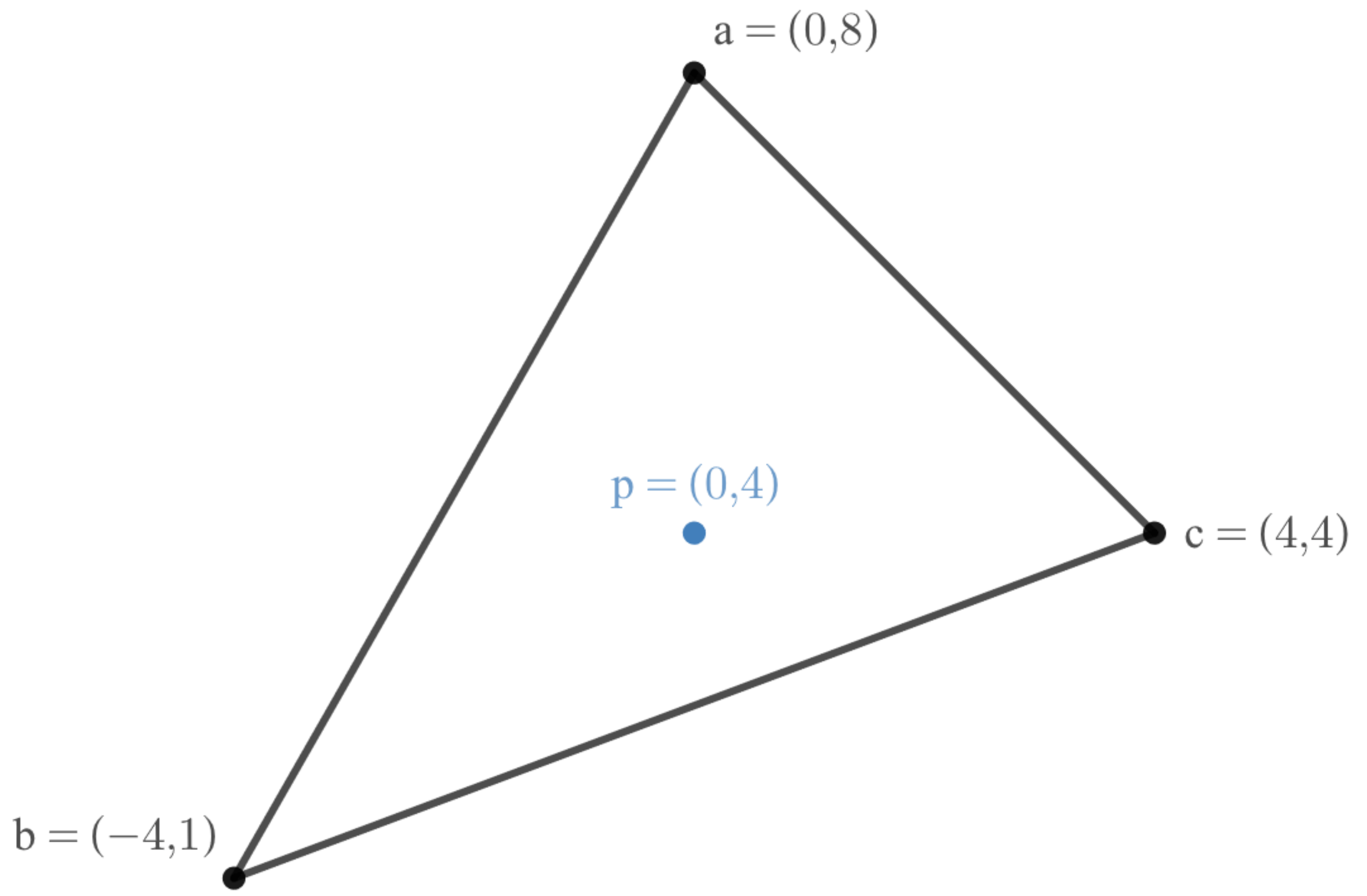
We set α, β, γ to be proportionate to the areas of the sub triangles formed by P.

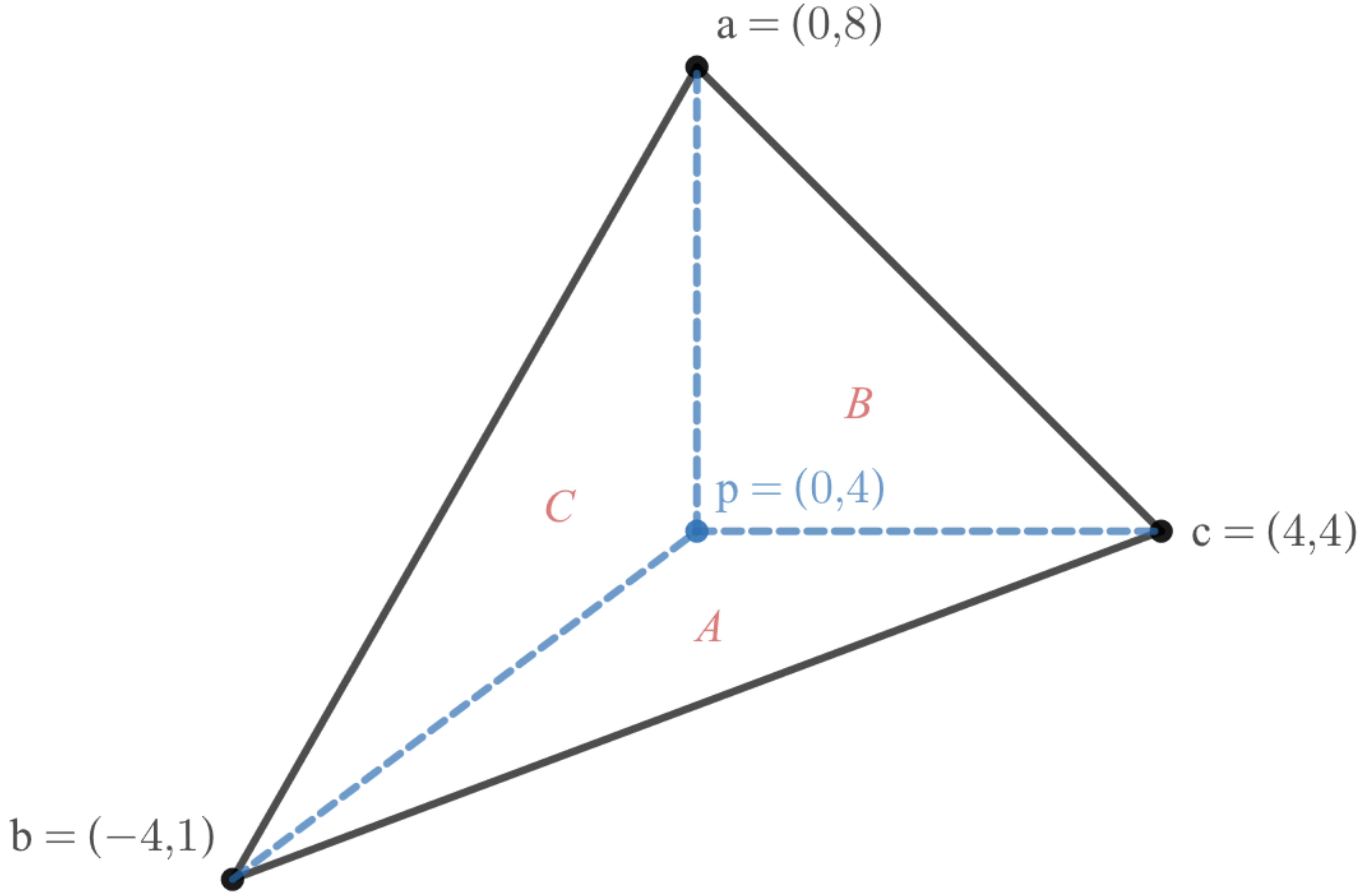
Thus:

$$\alpha = \frac{TriangleBCP_{area}}{TriangleABC_{area}}$$

$$\beta = \frac{TriangleCAP_{area}}{TriangleABC_{area}}$$

$$\gamma = \frac{TriangleABP_{area}}{TriangleABC_{area}}$$



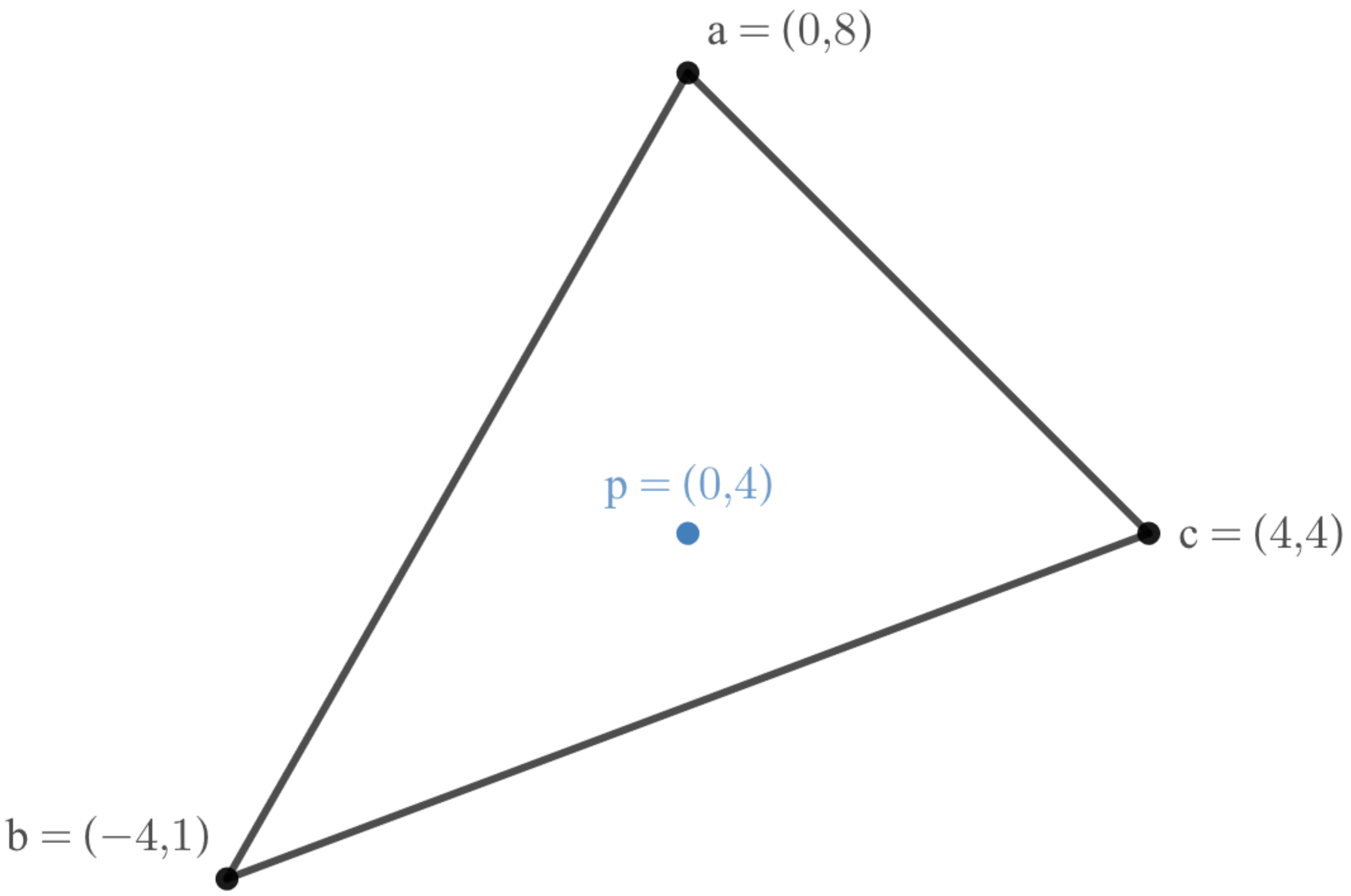


Barycentric coordinates

Exercise 1.

Find the barycentric coordinates (α, β, γ) of P in this example.

Hint: use cross product to find the area of each triangle.



Barycentric coordinates

$$A(A) = \frac{1}{2}\ \|(c-b)\times(p-b)\|\ = \frac{1}{2}\ \left\| \begin{bmatrix}8\\3\end{bmatrix} \times \begin{bmatrix}4\\3\end{bmatrix} \right\| = 6$$

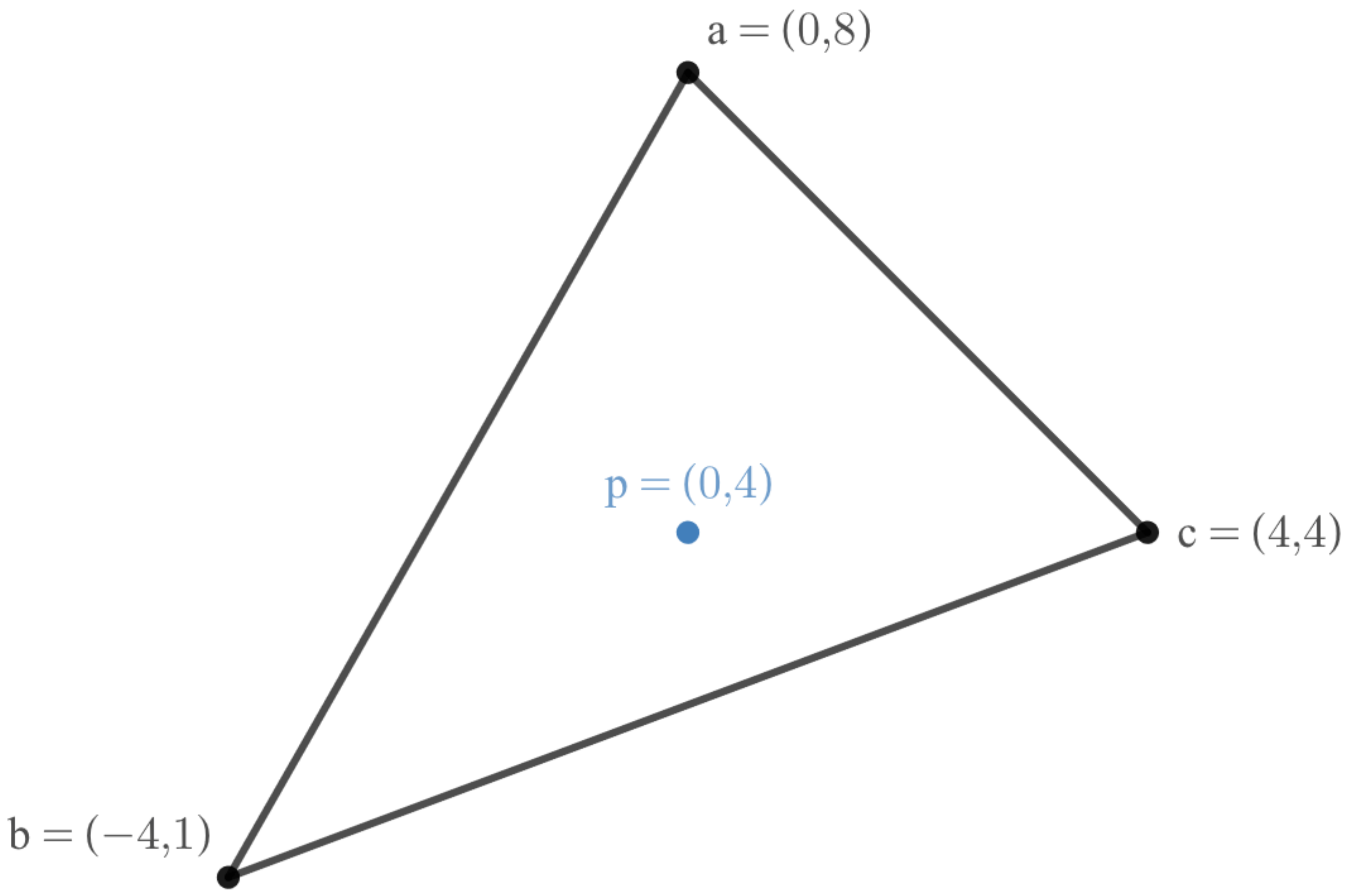
$$A(B) = \frac{1}{2}(4\cdot 4) = 8$$

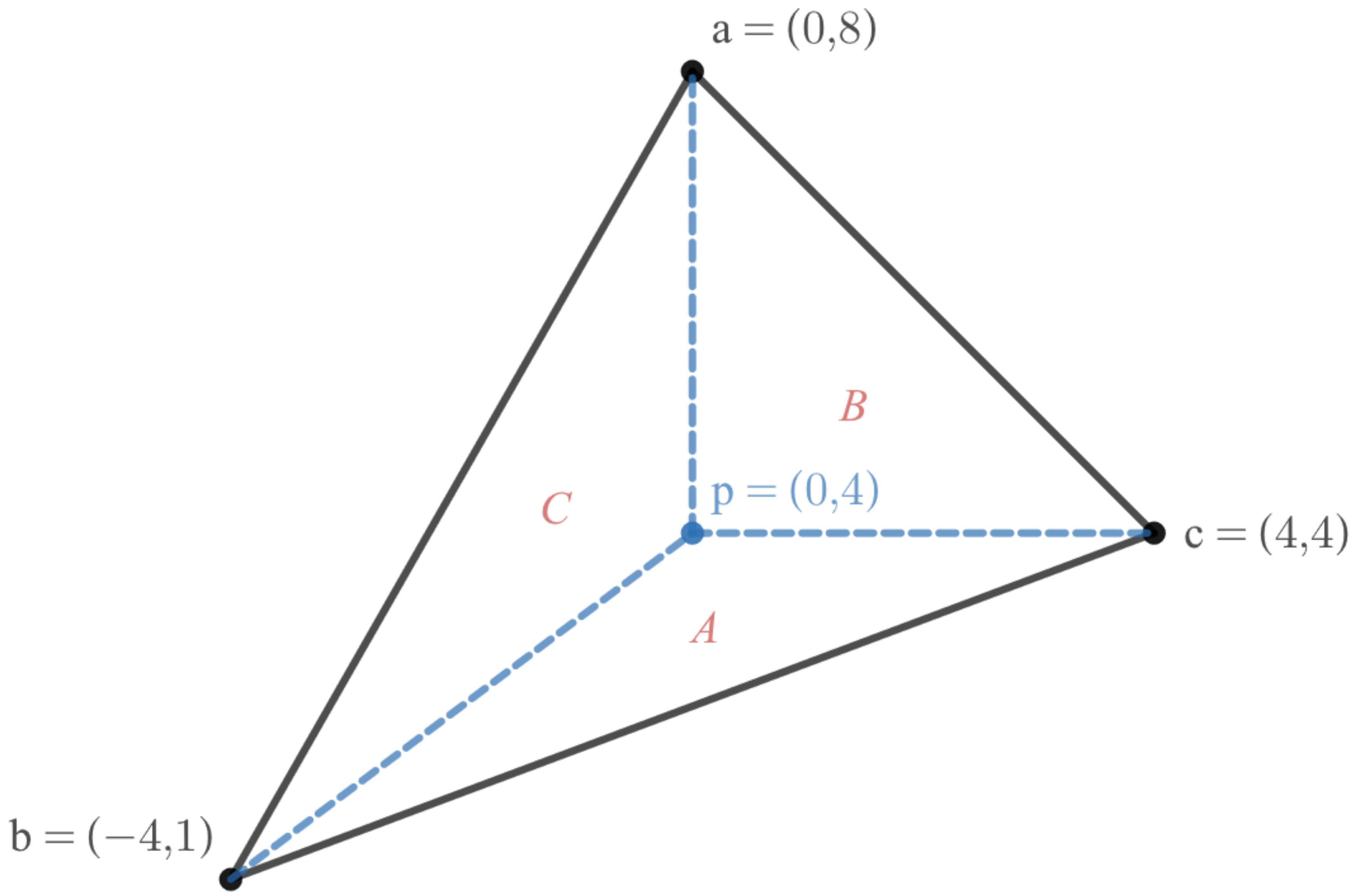
$$A(C) = \frac{1}{2}\ \|(a-b)\times(p-b)\|\ = \frac{1}{2}\ \left\| \begin{bmatrix}4\\7\end{bmatrix} \times \begin{bmatrix}4\\3\end{bmatrix} \right\| = 8$$

$$\alpha = \frac{6}{6+8+8} = \frac{3}{11}$$

$$\beta = \frac{8}{6+8+8} = \frac{4}{11}$$

$$\gamma = \frac{8}{6+8+8} = \frac{4}{11}$$



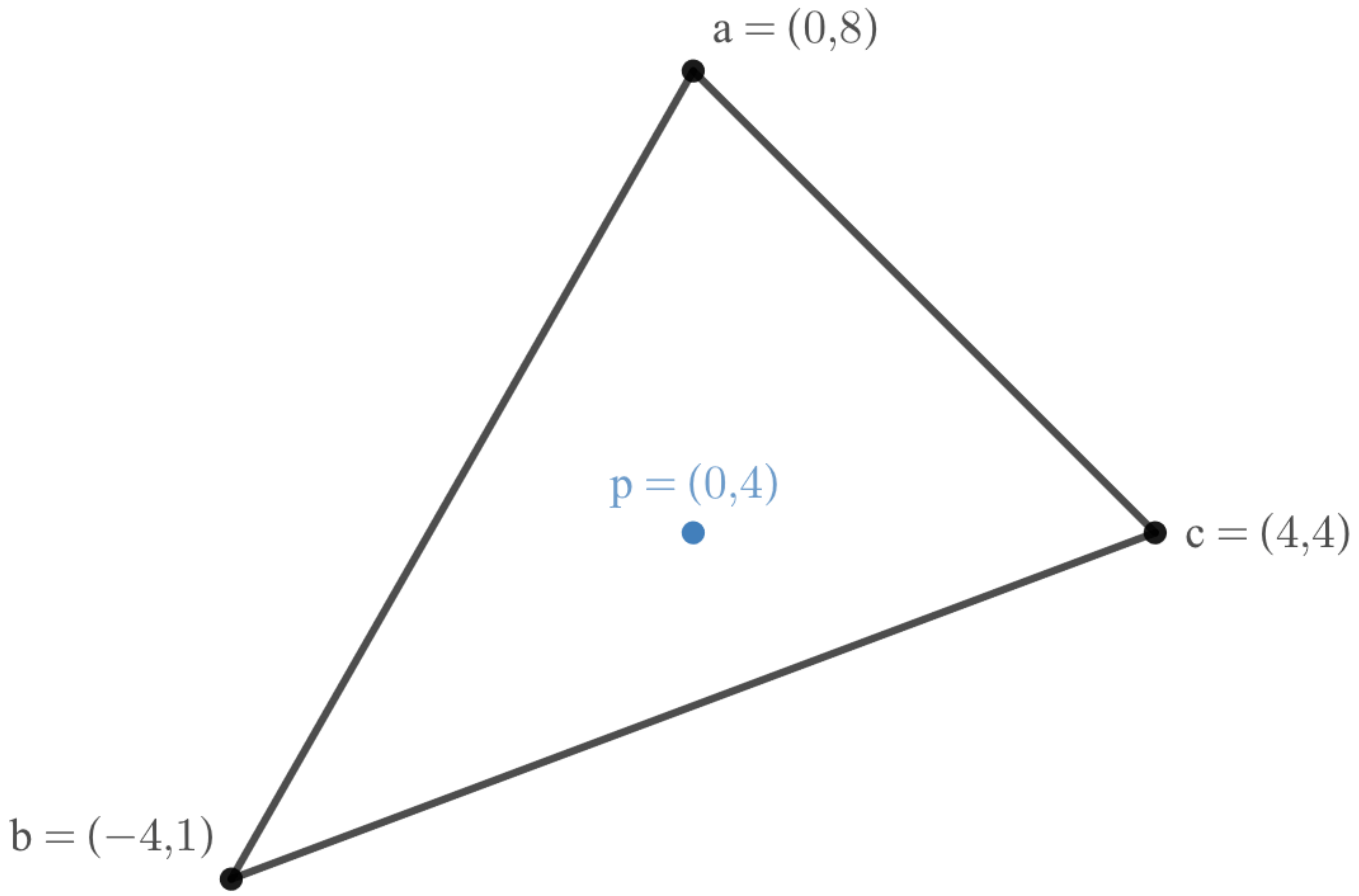


Barycentric coordinates

Exercise 2.

Construct a case where
one of α, β, γ is negative.
Draw out the resulting triangles
whose areas are proportional to
the barycentric coordinates.

In other words, where do you
move P in this figure?



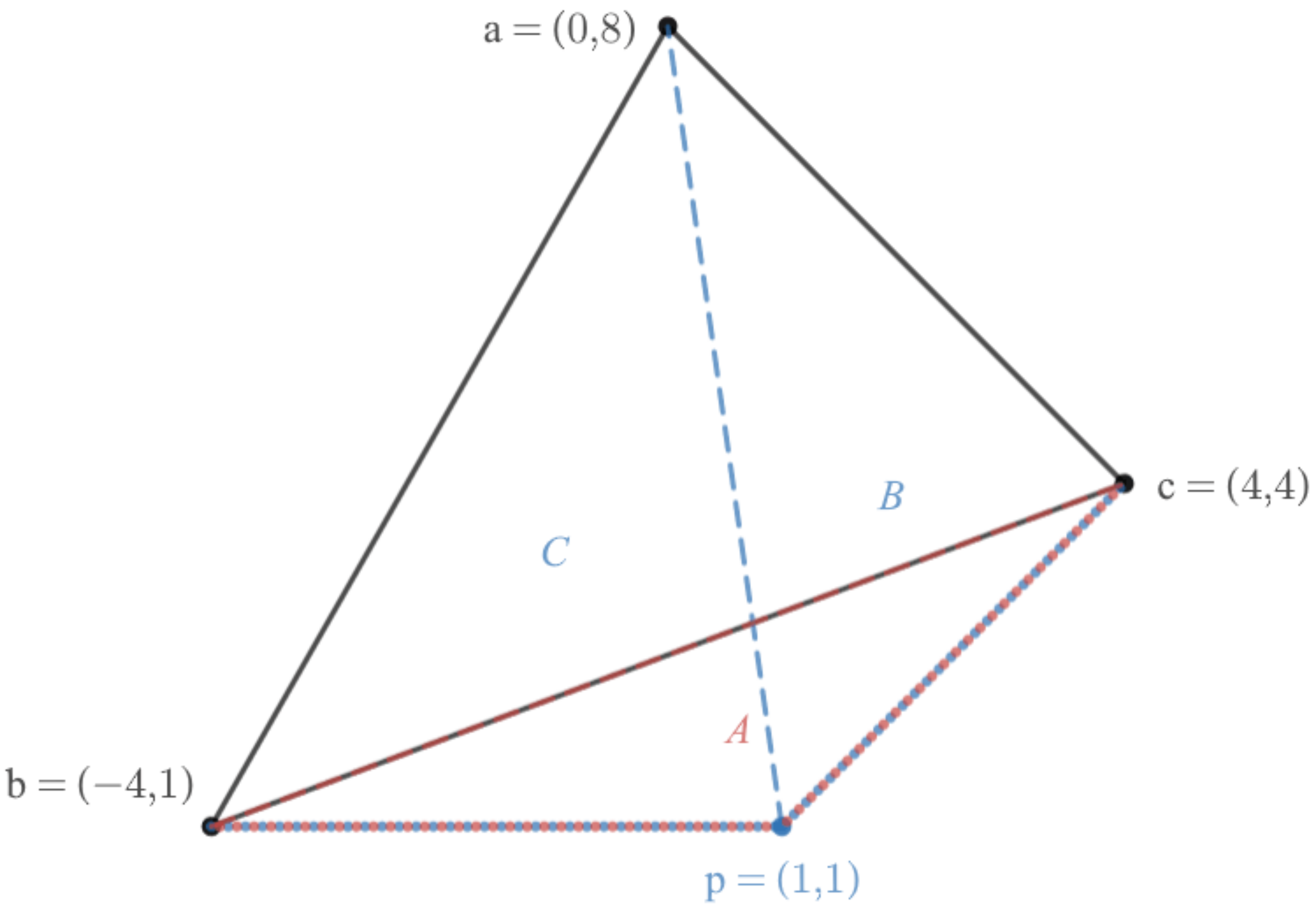
Barycentric coordinates

This is a possible solution where α is negative:

Note that the area of triangle A will be negative here.

This is because we are computing what is known as “signed area”.

The order of the vertices in triangle A changes when the area is positive/negative (the triangle is “flipped”).

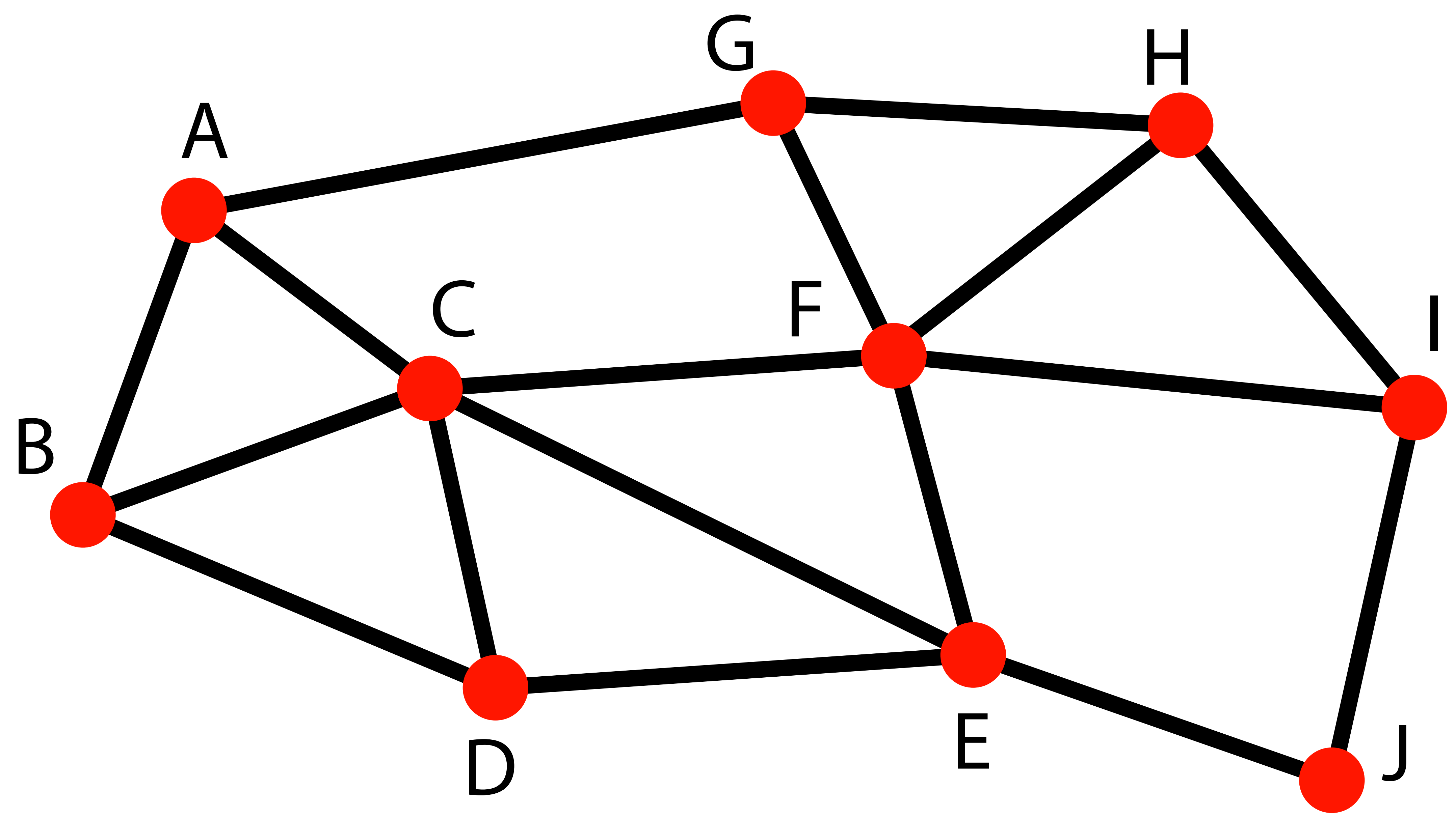


Standard Graphs

$G = \{V, E\}$

$V =$ vertices $= \{A, B, C, \ldots J\}$

$E =$ edges $= \{(A, B), (B, C), \ldots (I, J)\}$



Standard Graphs

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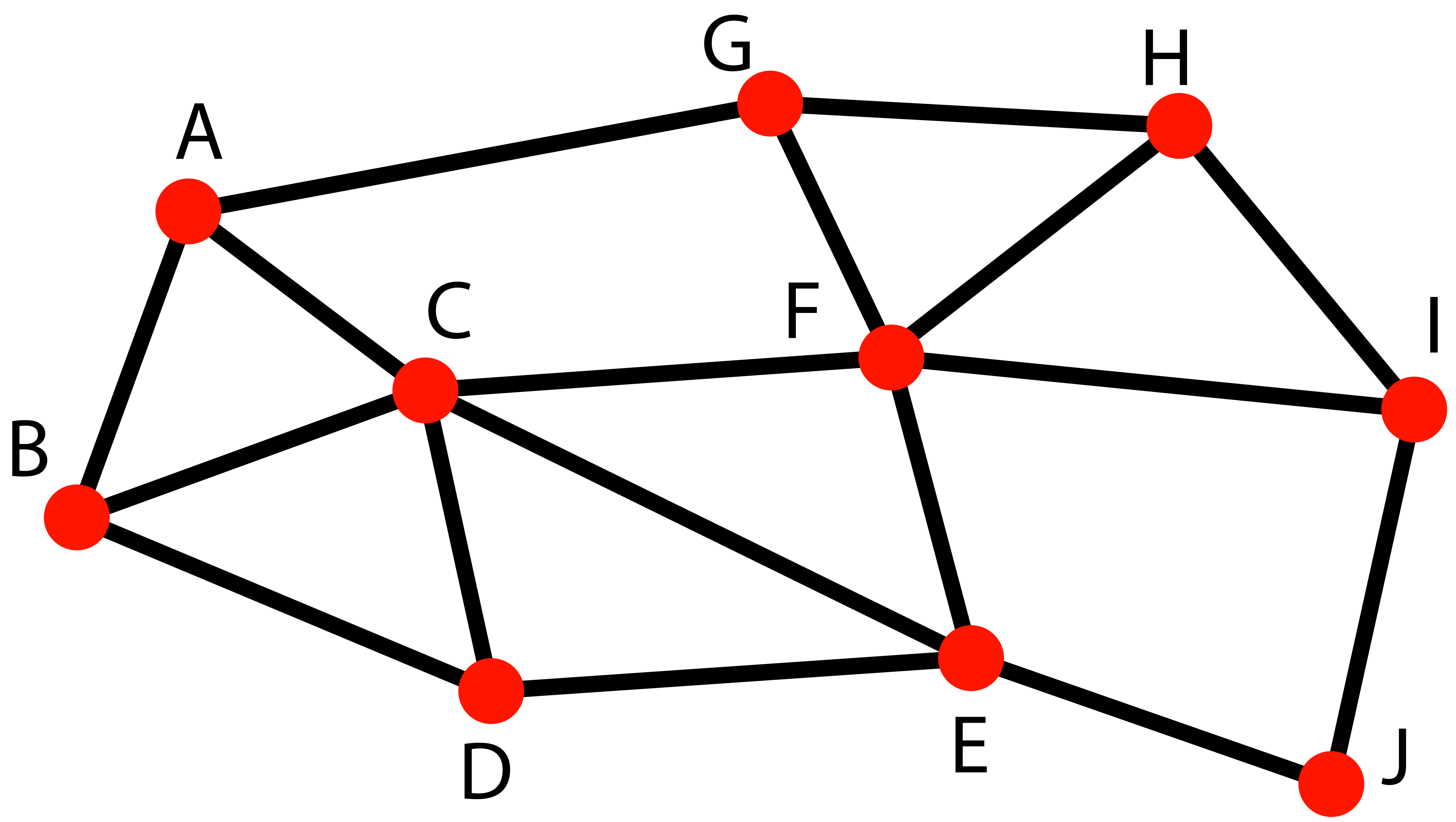
Face: cycle of edges that cannot be shortened.

$F =$ faces $= \{(A, B, C), (A, C, F, G), \ldots\}$



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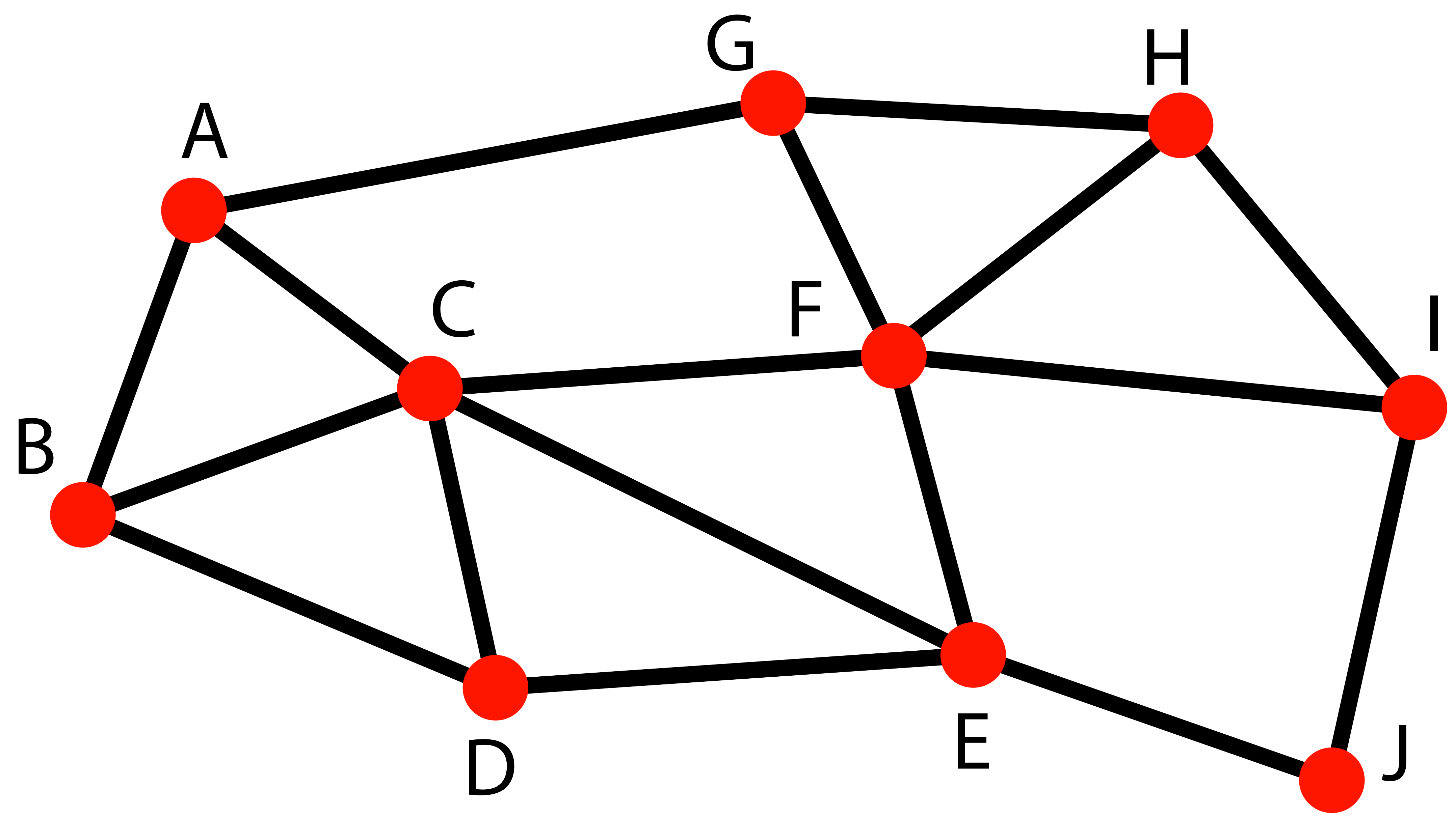
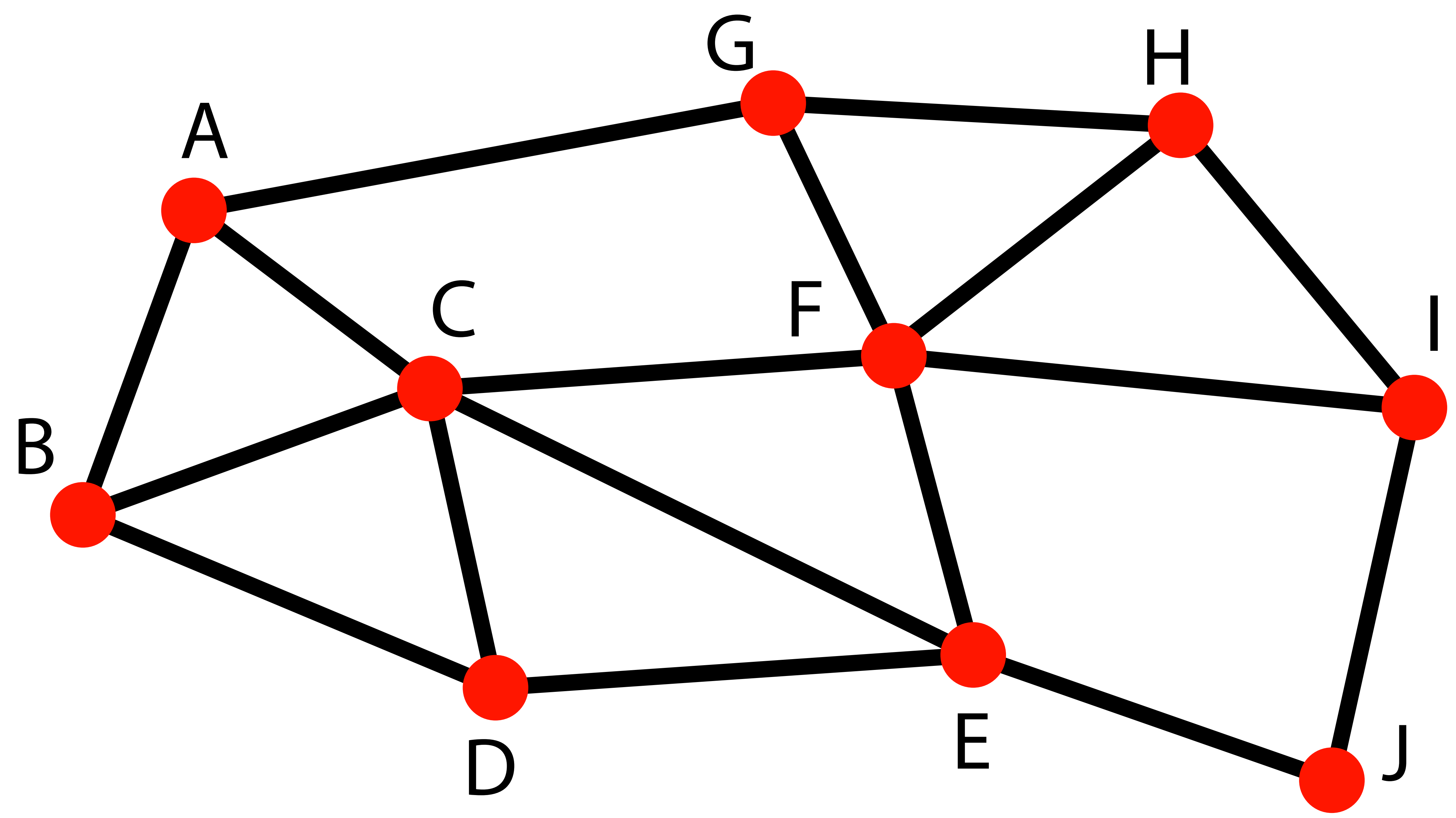
Degree (valence): number of edges incident on vertex (deg(A) = 3, deg(F) = 5)



Standard Graphs

A graph is **connected** if there is a path of edges that connect every two vertices.

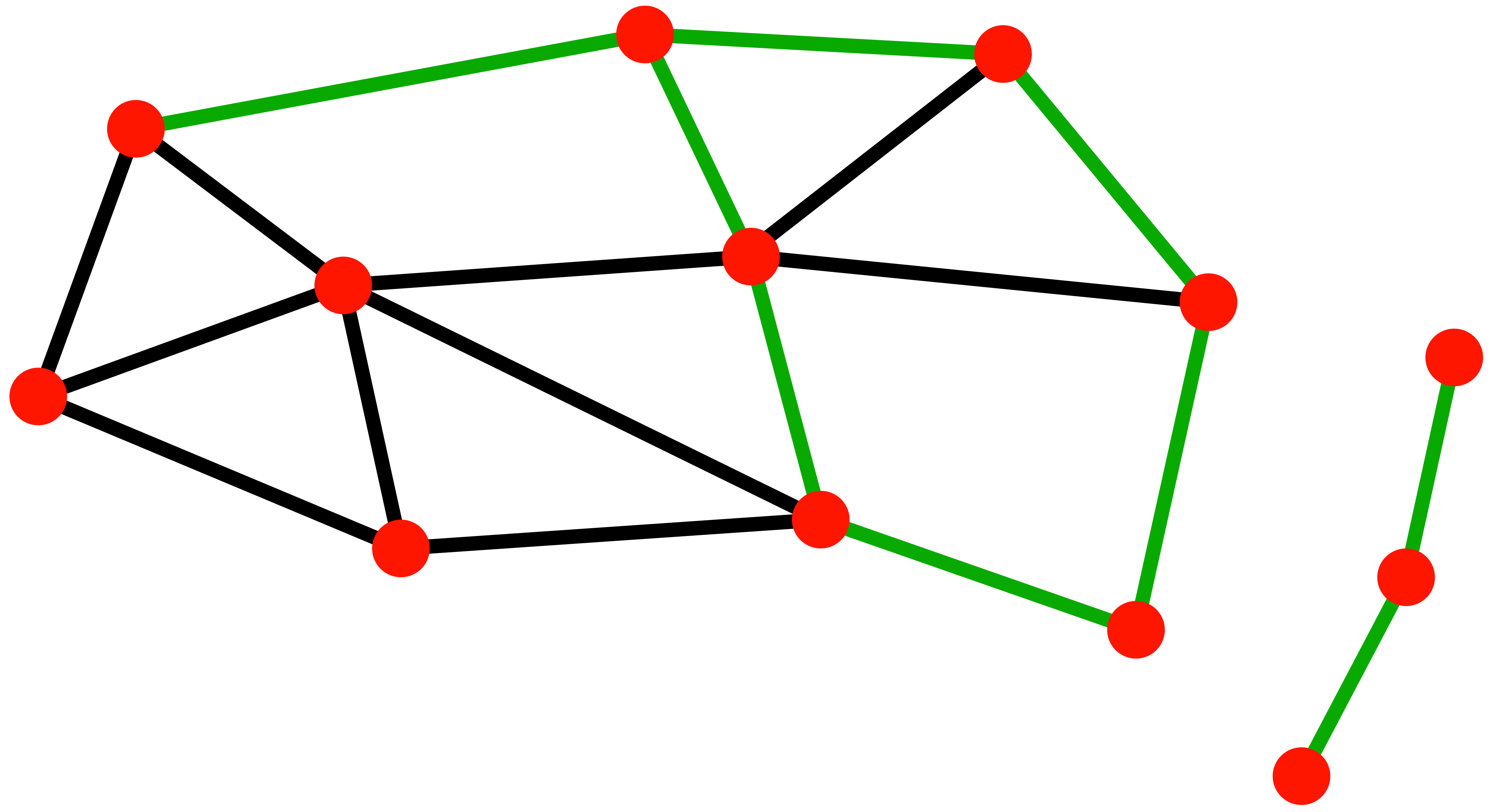


Standard Graphs

A graph is **connected** if there is a path of edges that connect every two vertices.

A graph $G' = (V', E')$ is a subgraph of $G = (V, E)$ if V' is a subset of V and E' is a subset of edges incident on V' (the two graphs in green are subgraphs of G)

A **connected component** of a graph is a maximally connected subgraph.

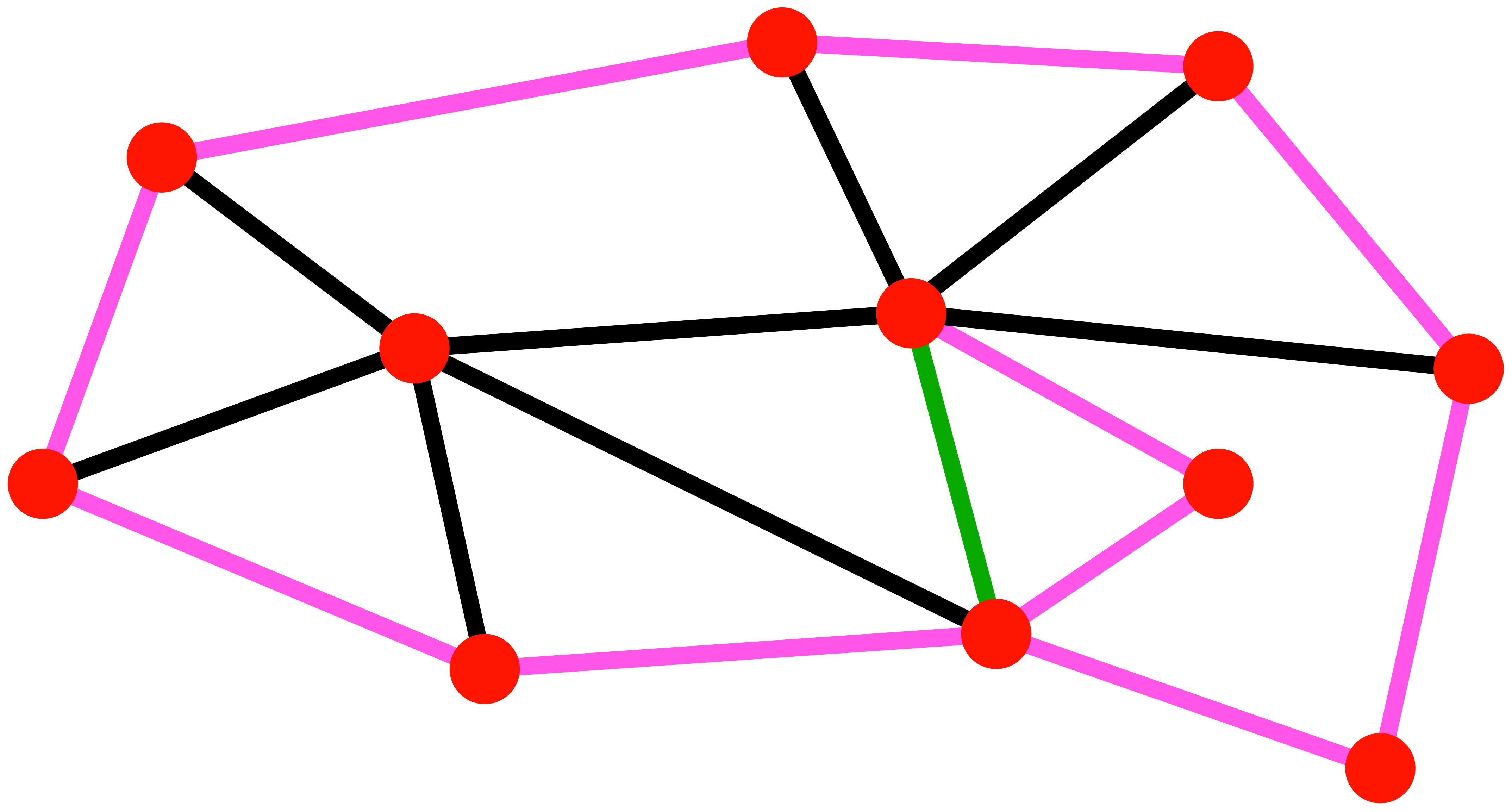


Standard Graphs

Boundary edge: adjacent to exactly one face (pink)

Regular edge: adjacent to exactly two faces (black)

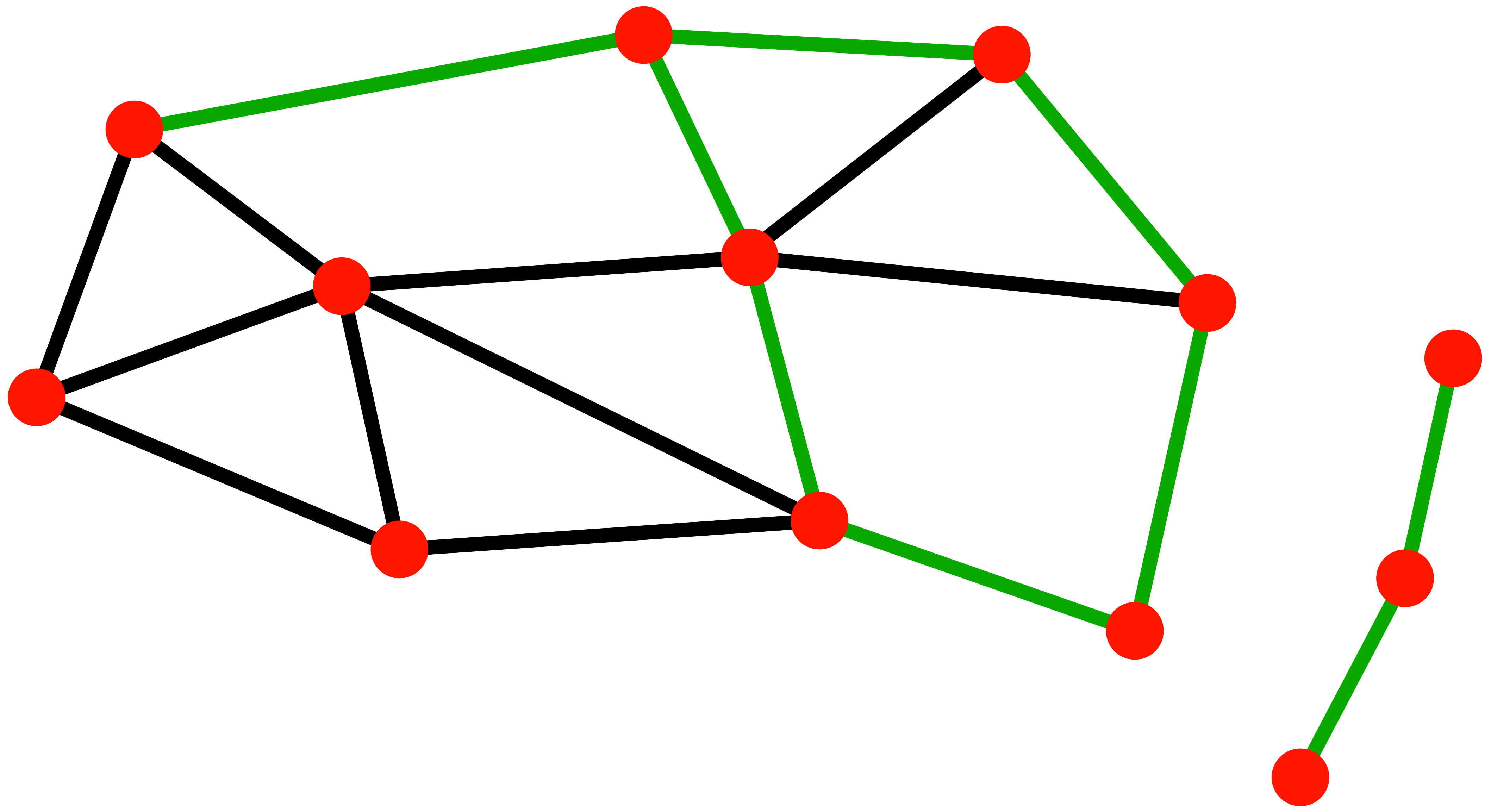
Singular edge: adjacent to more than two faces (green)



Standard Graphs

Exercise 3.

Give pseudo-code for an algorithm that, given an undirected graph $G = (V, E)$, returns the number of connected components in G.



Meshes

Mesh: a graph embedded in $\mathbb{R}^3$

We focus on triangular meshes - all faces have three vertices and three edges.

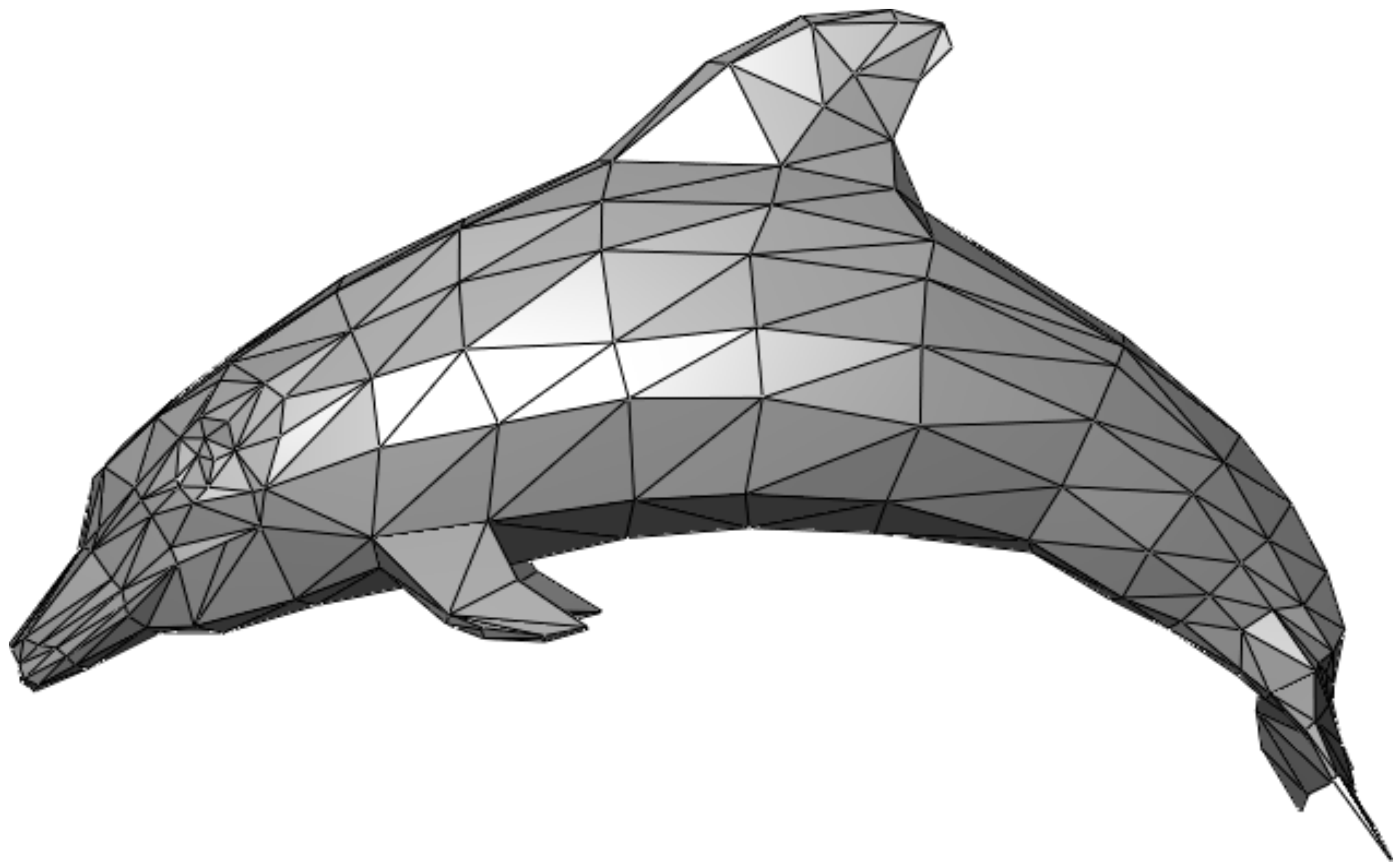
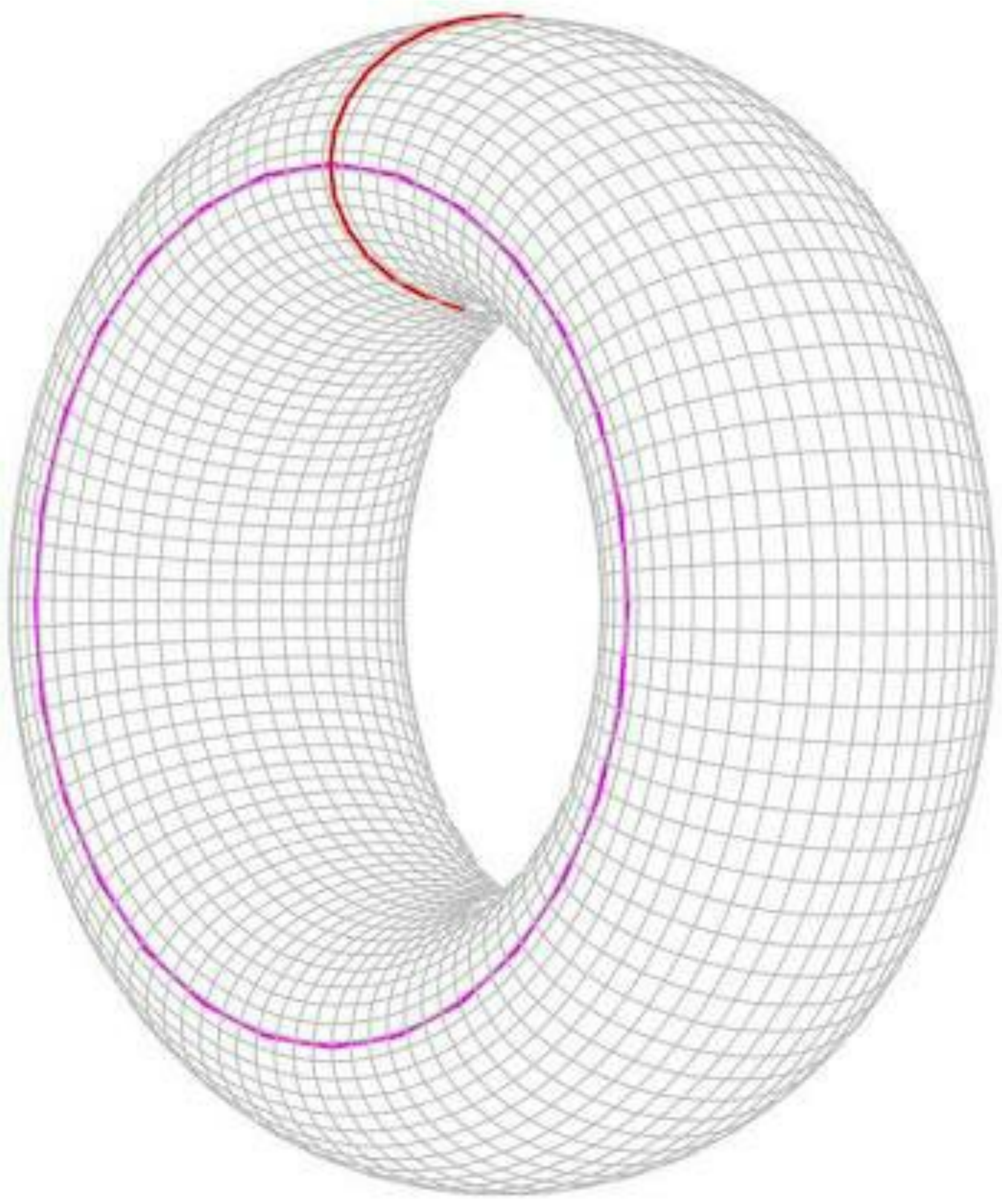


Image source: Wikipedia

Topology

Genus of graph: half of maximal number of closed paths that do not disconnect the graph (number of “holes”)

For a torus (donut shape), genus = 1
For a sphere, genus = 0



A torus and its two closed paths that do not disconnect the graph

Image source: Math Stack Exchange

Euler-Poincare Formula

$$v + f - e = 2(c - g) - b$$

v = # vertices

c = # conn. comp

f = # faces

g = genus

e = # edges

b = # boundaries

V = 10
F = 8
E = 17
C = 1
G = 0
B = 1