**Matlab Demo**

- Let’s pick out a signal
- Look at its Fourier coefficients
- Look at different combinations
  - Low frequencies vs. high frequencies
- I will put sample code up on the web if you want to experiment

**Alternative Description**

- Remember that
  \[
  e^{\sqrt{-1} \theta} = \cos \theta + \sqrt{-1} \sin \theta
  \]
  \[
  \cos \theta = \frac{e^{\sqrt{-1} \theta} + e^{-\sqrt{-1} \theta}}{2}
  \]
  \[
  \sin \theta = \frac{e^{\sqrt{-1} \theta} - e^{-\sqrt{-1} \theta}}{2\sqrt{-1}}
  \]

- So instead of having two sets of coefficients, for cosines and sines, make do with just one:
  \[
  F(i) = \sum_{j=-n/2+1}^{n/2} \hat{F}_j e^{\sqrt{-1}2\pi j(i/n)}
  \]

**Complex Fourier Coefficients**

- Now the Fourier transform of the signal (the coefficients \( \hat{F}_j \)) are complex numbers
  - Even if the signal was real!
- But can prove that pairs of coefficients are complex conjugates for real signals
  - Cuts storage in half, to what you expect
- Further trick - some codes rearrange coefficients so that
  \[
  F(i) = \sum_{j=0}^{n-1} \hat{F}_j e^{2\pi j(i/n)}
  \]

**Things to do with Fourier Series**

- Low-pass filter:
  - reduce or eliminate high frequency coefficients (gives smooth moving average)
- High-pass filter:
  - Reduce or eliminate low frequency coefficients (gives spiky details, independent of smooth baseline)
- Band-limiting (or band-pass) filter:
  - keep coefficients within a band of frequencies
- Compression:
  - Assume most high frequencies are small, can be thrown out unnoticeably (JPEG, MPEG)
- Understanding:
  - What frequencies are most active?
- Differential equations: may be simplified…
Things to worry about

- Fourier analysis assumes signal is periodic
  - But time isn’t: start and end usually don’t match up
    - Even periodic motion like walking may need some massaging to avoid implied discontinuity
- Comparing two signals: common time base
  - E.g. for motion, may want to do timewarping to match up features (or at least period for periodic motion)
- Lack of position information and global dependence
  - May want to look at windowed Fourier transform and/or wavelets