**Movable Objects**

- Before assumed $m_{\text{object}} >> m_{\text{particle}}$
  - Then effect on object is negligible
  - If not, still calculate new $v_{\text{rel}}$ as above
  - But change $v_{\text{object}}$ and $v_{\text{particle}}$ with “impulses”
- Unknown impulse $I$ (force * time) applied to particle and opposite $-I$ to object
- New velocities:
  - $v_{\text{particle}}^{\text{new}} = v_{\text{particle}} + \frac{I}{m_{\text{particle}}}$
  - $v_{\text{object}}^{\text{new}} = v_{\text{object}} - \frac{I}{m_{\text{object}}}$
- New relative velocity in terms of $I$ gives equation to solve for $I$:
  $$v_{\text{rel}}^{\text{new}} = v_{\text{rel}} + \left( \frac{1}{m_{\text{particle}}} + \frac{1}{m_{\text{object}}} \right) I$$

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**Friction**

- Friction slows down the relative tangential velocity
- Causes a tangential force $F_T$ that opposes sliding, according to
  - Magnitude of normal force $F_N$ pressing on particle
  - And friction coefficient $\mu$
- Basic Coulomb law:
  - If kinetic friction ($v_{\text{rel}} \neq 0$) then $|F_T| = \mu |F_N|$ and is in a direction most opposing sliding
  - If static friction ($v_{\text{rel}} = 0$) then $|F_T| \leq \mu |F_N|$

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**Implementing Friction**

- Gets really messy to directly use friction forces (really hard to get true static friction!)
- Instead integrate into relative velocity update
- Integrating normal and friction force over the collision time and dividing by mass gives Coulomb friction in terms of velocity changes:
  - Static friction: $|\Delta v_T| \leq \mu |\Delta v_N|$ (and then $v_T = 0$)
  - Kinetic friction: $\Delta v_T = -\mu |\Delta v_N| \frac{v_T}{|v_T|}$
    - Assuming direction of friction force is always opposing the initial tangential velocity
  - Combine into one formula for new relative tangential velocity:
    $$v_T^{\text{after}} = \max \left( 0, 1 - \frac{\mu |\Delta v_N|}{v_T^{\text{before}}} \right) v_T^{\text{before}}$$
Collisions so far

- We now have a black box collision processing routine
  - Input:
    - particle velocity before
    - (maybe object velocity and masses)
    - object normal
    - parameters $\varepsilon$ and $\mu$
  - Intermediate:
    - Relative velocity, split into normal and tangential components
  - Output:
    - new particle velocity
    - (maybe new object velocity)
- How do we use this in time integration?

Simple collision algorithm

- After each time step, check if particles collided with objects
  - If so, change velocities according to routine
- Fails catastrophically for more interesting cases
  - New velocity may or may not get particle out next time step - is that another collision?
  - Is it ok to have particles inside (or on the wrong side of) objects any time?

Backing up time

- Can avoid some problems by processing collision when it happens, not after the fact
- Figure out when collision happens (or at least get close to time of collision, but not later than)
- Apply velocity update then
- Potential problems:
  - Hard to figure out time
  - Could involve a lot of work per time step (unpredictable)

Simultaneous collision resolution

- Ignore exact timing and order of collisions during a time step
- Begin with old position $x^{\text{old}}$
- New candidate position $x^{\text{new}}$
- If collision occurred, process with $v^{\text{avg}} = (x^{\text{new}}-x)/\Delta t$ to get new post-collision velocity $v^{after}$
- Then change $x^{\text{new}}$ to $x^{after} = x^{\text{old}} + \Delta t \cdot v^{after}$
- Iterate until no collisions remain
Notes on collision resolution

- This works really well for inelastic collisions
- Can use a large $\Delta t$: separate collision processing from particle physics
  - Can take many small steps to from $x_{\text{old}}$ to $x_{\text{new}}$ if stability demands it
- Problems arise with elastic collisions
  - May not converge
  - Bouncing block problem: a block won’t come to rest on the floor

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Elastic collision resolution

- Start with $x_{\text{old}}$ and $v_{\text{old}}$
- Advance to $x_{\text{new}}$
- If collision, apply elastic collision law to $v_{\text{old}}$ to get $v^2$
- Take $x^2 = x_{\text{new}} + \Delta t (v^2 - v_{\text{old}})$ or reintegrate from $x_{\text{old}}$, $v^2$ if you can afford it
  - Repeat elastic step a few times if you want, and there are still collisions with $x^2$
- If still collision, apply INELASTIC collision law to $v_{\text{avg}} = (x^2 - x_{\text{old}})/\Delta t$ to get $v_{\text{after}}$
- Change $x^2$ to $x_{\text{after}} = x_{\text{old}} + \Delta t v_{\text{after}}$
- Repeat as needed

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One last problem

- Due to round-off error, or pathological geometry, may still go into a long loop resolving collisions
- So cut loop off after a small number of iterations
- Failsafe: take $v_{\text{after}} = 0$, $x_{\text{new}} = x_{\text{old}}$
  - May look weird, still could have issues for moving objects especially
- Last resort: accept the penetration, apply a repulsion force to eventually move the object out from the object(s)